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Influence of convective cooling on the temperature in a frictionally heated strip and foundation ☆

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ABSTRACT

The analytical solution of a boundary-value heat conduction problem of friction for the tribosystem consisting of a semi-infinite foundation and a plane-parallel strip sliding over its surface is obtained. The evolution of temperature and its distribution in depth from a contact surface for materials of frictional couple, such as aluminum-steel, is studied.

1. Introduction

The heat conduction problems of friction are nowadays formulated in two variants. In the first one, the elements of friction couple are considered separately and the heat flow intensities are set on each of the contact surfaces in such a way that their sum equals the specific power of friction [1–5]. For this purpose, the heat participation factor is introduced which is found experimentally or by empirical formulas [6–8]. It is the statement that gives the solution of the heat conduction problem of friction for the foundation with a composite strip [9,10] and the heat conduction problem for the braking of a massive body coated with either a homogeneous [11] or a composite [12] strip. The thermoelastic state resulted from the heating of the piecewise-homogeneous body consisting of a semi-infinite foundation and a strip by the heat pulse of a finite duration is studied in article [13].

Another variant of the statement of heat conduction problems of friction is based on the simultaneous solution of the heat conduction equations for both friction elements followed by the determination of the heat flows intensities on their heating [14–18]. In such statement the problems of transient frictional heating in cold rolling of metals [19], the heat transfer in friction welding of cylindrical rods with different diameters [20] and the fast-moving heating on the external surface of the ring due to friction of two rotating pins [21] were analyzed. The analytical solution of a boundary-value problem of heat conduction for tribosystem, consisting of the homogeneous semi-space, sliding uniformly on a surface of the strip deposited on a semi-infinite substrate, was obtained in paper [22].

The aim of the study is to obtain the solution of the transient problem of heat conduction for the tribosystem, consisting of a strip sliding over the surface of a semi-infinite foundation at a constant velocity.

2. Statement of the problem

The problem of contact interaction of a plane-parallel strip and semi-infinite foundation (the semi-space) is considered. It is supposed, that the constant compressive pressures p_0 in the direction of the z-axis of the Cartesian system of coordinates $Oxyz$ are applied to the upper surface of a strip and to the infinity in semi-space (Fig. 1). The strip slides with the constant velocity V in the direction of the y-axis on the semi-space surface. Due to friction the heat is generated on a contact plane $z=0$. The sum of the intensities of the frictional heat fluxes directed into each component of friction pair is equal to the specific friction power $q_0=fVp_0$ [15]. Contacting surfaces of a strip and the foundation are smooth. Therefore, the contact temperatures of a strip and foundations are equal. The strip surface $z=d$ is under condition of convective heat exchange with the surrounding. Let us find the distribution of temperature fields in the strip and in foundation. Further, all values and the parameters concerning a strip and foundation will have bottom indexes “s” and “f ”, respectively.

The transient temperature fields in the strip $T_s(z,t)$ and in the foundation $T_f(z,t)$ can be found from the solution of the heat conduction problem of friction:

$$\frac{\partial^2 T_s(z,t)}{\partial z^2} = \frac{1}{k_s}\frac{\partial T_s(z,t)}{\partial t}, \qquad 0<z<d,\ t>0, \tag{1}$$

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Nomenclature

d	Thickness of the strip
$\mathrm{erf}(x)$	Gauss error function
$\mathrm{erfc}(x)=1-\mathrm{erf}(x)$	Complementary error function
$\mathrm{ierfc}(x)=\pi^{1/2}\exp(-x^2)-x\,\mathrm{erfc}(x)$	Integral of the error function $\mathrm{erfc}(x)$
f	Frictional coefficient
h_s	Heat transfer coefficient
K	Thermal conductivity
k	Thermal diffusivity
p_0	Pressure
$q_0=fVp_0$	Specific friction power
T	Temperature
$T_0=q_0d/K_s$	Temperature scaling factor
$T^*=T/T_0$	Dimensionless temperature
t	Time
V	Sliding velocity
z	Spatial coordinate

Greek symbols

$Bi_s=h_sd/K_s$	Biot's number
$\tau=k_st/d^2$	Fourier's number
$\zeta=z/d$	Dimensionless spatial coordinate

Indexes

f	Foundation
s	Strip

$$\frac{\partial^2 T_f(z,t)}{\partial z^2}=\frac{1}{k_f}\frac{\partial T_f(z,t)}{\partial t},\qquad 0<z<d,\ t>0, \tag{2}$$

$$K_f\frac{\partial T_f}{\partial z}\Big|_{z=0-}-K_s\frac{\partial T_s}{\partial z}\Big|_{z=0+}=q_0,\quad t>0, \tag{3}$$

$$T_f(0,t)=T_s(0,t),\quad t>0, \tag{4}$$

$$K_s\frac{\partial T_s}{\partial z}\Big|_{z=d}+h_sT_s(d,t)=0,\quad t>0, \tag{5}$$

$$T_f(z,t)\to 0,\ z\to-\infty,\ t>0, \tag{6}$$

$$T_s(z,0)=0,\quad 0\le z\le d, T_f(z,0)=0,\quad -\infty<z\le 0. \tag{7}$$

Let us denote by

$$\zeta=\frac{z}{d},\quad \tau=\frac{k_st}{d^2},\quad Bi_s=\frac{h_sd}{K_s},\quad K^*=\frac{K_f}{K_s},\quad k^*=\frac{k_f}{k_s},\quad T_0=\frac{q_0d}{K_s},\quad T^*_{s,f}=\frac{T_{s,f}}{T_0}. \tag{8}$$

Taking above expressions (8) into account, the boundary-value problem of heat conduction Eqs. (1)–(7) can be written down in the form

$$\frac{\partial^2 T^*_s(\zeta,\tau)}{\partial \zeta^2}=\frac{\partial T^*_s(\zeta,\tau)}{\partial \tau},\quad 0<\zeta<1,\ \tau>0, \tag{9}$$

$$\frac{\partial^2 T^*_f(\zeta,\tau)}{\partial \zeta^2}=\frac{1}{k^*}\frac{\partial T^*_f(\zeta,\tau)}{\partial \tau},\quad -\infty<\zeta<0,\ \tau>0, \tag{10}$$

$$K^*\frac{\partial T^*_f}{\partial \zeta}|_{\zeta=0-}-\frac{\partial T^*_s}{\partial \zeta}|_{\zeta=0+}=1,\quad \tau>0, \tag{11}$$

$$T^*_f(0,\tau)=T^*_s(0,\tau),\quad \tau>0, \tag{12}$$

$$\frac{\partial T^*_s}{\partial \zeta}|_{\zeta=1}+Bi_sT^*_s(1,\tau)=0,\quad \tau>0, \tag{13}$$

$$T^*_f(\zeta,\tau)\to 0,\quad \zeta\to-\infty,\tau>0, \tag{14}$$

$$T^*_s(\zeta,0)=0,\quad 0\le\zeta\le 1,\quad T^*_f(\zeta,0)=0,\quad -\infty<\zeta\le 0. \tag{15}$$

3. Solution of the problem

By applying the Laplace integral transform to the Eqs. (9)–(15) with respect to the dimensionless time τ [23]

$$L[T_{s,f}(\zeta,\tau);p]\equiv\overline{T}^*_{s,f}(\zeta,p)=\int_0^\infty T^*_{s,f}(\zeta,\tau)e^{-p\tau}d\tau, \tag{16}$$

we obtain

$$\frac{d^2\overline{T}^*_s(\zeta,p)}{d\zeta^2}-p\overline{T}^*_s(\zeta,p)=0,\quad 0<\zeta<1, \tag{17}$$

$$\frac{d^2\overline{T}^*_f(\zeta,p)}{d\zeta^2}-\frac{p}{k^*}\overline{T}^*_f(\zeta,p)=0,\quad -\infty<\zeta<0, \tag{18}$$

$$K^*\frac{d\overline{T}_f}{d\zeta}|_{\zeta=0-}-\frac{d\overline{T}_s}{d\zeta}|_{\zeta=0+}=\frac{1}{p}, \tag{19}$$

$$\overline{T}^*_f(0,p)=\overline{T}^*_s(0,p), \tag{20}$$

$$\frac{d\overline{T}^*_s(\zeta,p)}{d\zeta}|_{\zeta=1}+Bi_s\overline{T}^*_s(1,p)=0, \tag{21}$$

$$\overline{T}^*_f(\zeta,p)\to 0,\quad \zeta\to-\infty. \tag{22}$$

The solutions of the ordinary differential Eqs. (17) and (18) at boundary conditions (19)–(22) have the form:

$$\overline{T}^*_s(\zeta,p)=\frac{\Delta_s(\zeta,p)}{p\sqrt{p}\Delta(p)},\quad 0\le\zeta\le 1, \tag{23}$$

$$\overline{T}^*_f(\zeta,p)=\frac{\Delta_f(\zeta,p)}{p\sqrt{p}\Delta(p)},\quad -\infty<\zeta\le 0, \tag{24}$$

where

$$\Delta_s(\zeta,p)=\sqrt{p}ch[(1-\zeta)\sqrt{p}]+Bi_ssh[(1-\zeta)\sqrt{p}], \tag{25}$$

$$\Delta_f(\zeta,p)=[\sqrt{p}ch(\sqrt{p})+Bi_ssh(\sqrt{p})]e^{\zeta\sqrt{\frac{p}{k^*}}}, \tag{26}$$

$$\Delta(p)=(\sqrt{p}+\varepsilon Bi_s)sh(\sqrt{p})+(\varepsilon\sqrt{p}+Bi_s)ch(\sqrt{p}), \tag{27}$$

$$\varepsilon=\frac{K_f}{K_s}\sqrt{\frac{k_s}{k_f}}. \tag{28}$$