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On the transition for a generalized Couette flow of a reactive third-grade fluid with viscous dissipation☆

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Abstract

We study the thermal transition of a reactive flow of a third-grade fluid with viscous heating and chemical reaction between two horizontal flat plates, where the top is moving with a uniform speed and the bottom plate is fixed in the presence of imposed pressure gradient. This study is a natural continuation of earlier work on rectilinear shear flows. The governing equations are non-dimensionalized and the resulting system of equations are not coupled. An approximate explicit solution is found for the flow velocity using homotopy-perturbation technique and the range of validity is determined. After the velocity is known, the heat transport may be analyzed. It is found that the temperature solution depends on the non-Newtonian material parameter of the fluid, Λ, viscous heating parameter, Γ, and an exponent, m. Attention is focused upon the disappearance of criticality of the solution set $\{\beta, \delta, \theta_{\max}\}$ for various values of Λ, Γ and m, and the numerical computations are presented graphically to show salient features of the solution set.

Keywords: Generalized Couette flow; Fluid of third grade; Homotopy perturbation; Variable pre-exponential factor

1. Introduction

Considerable attention has been given to generalized Couette flow, in part because it is a patterned example for diverse phenomena and industrial applications. Generalized Couette flow occurs when a fluid is placed between two long parallel flat plates, where the top plate is moving in its own plane with a constant speed and the bottom plate is fixed in the presence of externally imposed pressure gradient [24]. This flow corresponds to an ideal and limiting case of the flow between concentric rotating cylinders (see, [9,5]). There are many practical applications in [8] and further references cited there-in.

For unidirectional Newtonian fluid with first-order reaction, Adler [1] studied criticality for steady developed reactive flow between symmetrically heated parallel walls while Zaturska [29] investigated criticality for the reactive plane Couette flow. Studies by Shonhiwa and Zaturska [25,26] examined transitional values for each of the reactive flows in the aforementioned under physically reasonable assumptions. We should mention that the generalized Couette flow was not investigated, in part because the resulting equation for the velocity field is linear and solution to the problem is a superposition of the simple Couette flow over the plane Poiseuille flow.

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The shearing motion for channel flow using other constitutive relations, has been studied by many authors ([2,3,8,12,14,17,23,30]). Recently, Hayat et al. [13] found series solution using homotopy analysis method for Oldroyd 6-constant fluid in generalized Couette device.

In particular, the steady state equation of motion of incompressible Newtonian fluid and the third grade fluids are both second-order ordinary differential equations. The marked difference between the case of the Navier Stokes theory and that for fluids of third-grade is that ignoring the nonlinearity in the case of the third-grade fluids, reduces to Newtonian fluids. Thus, in this special case, the third-grade fluid is a generalized Newtonian fluid. The aforementioned third-grade fluid motion has been analyzed by [4,10,15,22,23,27,28]. It can be easily showed that if the pressure gradient is dropped from the third-grade flow, the profile for the simple Couette flow is recovered. However, since the third-grade fluid is nonlinear in the velocity and the principle of superposition being not applicable, the generalized Couette device should be treated as an individual problem.

Third-grade fluid with the generation of chemical reaction heat inside the slab has recently been investigated. In [20] we have shown for the plane Poiseuille flow that the nonlinear effects from the velocity and temperature fields introduced decaying for the transitional values of the dimensionless central temperature. Criticality and transition for a steady reactive simple Couette flow of a viscous fluid have also been obtained in [21] using numerical method. Hermite-Pade approximation technique has been used in [16] to investigate the bifurcations of steady flow through a cylindrical pipe with isothermal wall. In fact, what makes one flow situation different from another is the boundary conditions, rheological properties of the type of flow and physicochemical parameters, such as exothermicity and reactivity.

The goal of this paper is therefore to investigate the system of equations for a reactive viscous flow of an incompressible, homogeneous fluid of third-grade in a generalized Couette device. We provide an analytical framework using a homotopy-perturbation technique in place of perturbation method in evaluating the velocity field. We will use this velocity field in the temperature equation and then study the thermal transition. Comparative evaluation of the non-Newtonian material parameter in this study allow us to confirm that the phenomenon cannot be neglected.

2. The physical and mathematical model

Using the dimensionless variables u, θ, β, Λ, Γ and y of the new variable approach of Okoya [20,21] and the references contained therein, the steady hydrodynamically and thermally developed unidirectional flow of a reactive third grade fluid with viscous dissipation between two parallel plates that are infinite in the $\bar{x}$-direction, located in the $\bar{y}=-\bar{y}_0$ and $\bar{y}=\bar{y}_0$, respectively and wide enough in the $\bar{z}$-direction to have negligible side effects can be written as

$$\frac{d^2u}{dy^2}\left(1+6\Lambda\left(\frac{du}{dy}\right)^2\right)=C, \tag{1}$$

$$\frac{d^2\theta}{dy^2}+\Gamma\left(\frac{du}{dy}\right)^2\left(1+2\Lambda\left(\frac{du}{dy}\right)^2\right)+\delta(1+\beta\theta)^m\exp\left(\frac{\theta}{1+\beta\theta}\right)=0, \tag{2}$$

$$u(-1)=0, u(1)=1, \tag{3}$$

$$\theta(-1)=0, \theta(1)=0, \tag{4}$$

where

$$u=\frac{\bar{u}}{\bar{U}_0}, \beta=\frac{R\bar{T}_0}{E}, \Gamma=\frac{\mu\bar{U}_0^2}{K\bar{T}_0\beta}, \theta=\frac{(\bar{T}-\bar{T}_0)E}{R\bar{T}_0^2}, C=\frac{\bar{y}_0^2}{\mu\bar{U}_0}\frac{dP}{d\bar{x}},$$

$$y=\frac{\bar{y}}{\bar{y}_0}, \Lambda=\frac{\beta_3\bar{U}_0^2}{\mu\bar{y}_0^2}, \delta=\frac{QEA_0\bar{y}_0^2C_0k^m\bar{T}_0^{m-2}}{v^m\hbar^m RK}\exp\left(\frac{E}{R\bar{T}_0}\right),$$

in the absence of body forces, neglecting reactant consumption and assuming generalized reaction rate law as well as Fourier's law of heat conduction with uniform thermal conductivity of the material. Here Γ is the viscous heating parameter, δ is the Frank–Kamenetskii parameter, θ is the dimensionless temperature excess, u is the dimensionless velocity, β is the activation energy, Λ is the non-Newtonian material parameter of the fluid and y is the fractional distance from the central plane.