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ScienceDirect
International Communications in Heat and Mass Transfer 35 (2008) 204 – 214
International Communications in HEAT and MASS TRANSFER
www.elsevier.com/locate/ichmt

Variational formulation of hyperbolic heat conduction problems applying Laplace transform technique ☆

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Available online 14 August 2007

Abstract

In this paper, a non-Fourier heat conduction problem is analyzed by employing newly developed theory. Application of conventional numerical schemes leads to strong oscillations of the results around discontinuities in solution domain. To overcome this difficulty the variational formulation of the Laplace-transformed hyperbolic heat conduction equation is developed. The results were used for evaluation of parameters used in approximate transformed temperature profiles. To validate the approach the results were compared with the exact analytical solution solved at special case and with an approach previously reported in the literature. Both showed a close agreement with the proposed approach.

Keywords: Hyperbolic heat conduction equation; Variational formulation; Laplace transform

1. Introduction

Biot has developed a new method in heat flow analysis [1], he has given a new form to the variational formulation of Fourier heat conduction equation, and has drawn attention to the similarity of this formulation to that of mechanics. In this connection, he has introduced the concept of penetration depth and penetration time for one-dimensional transient problems (plates). Ordinary but nonlinear differential equation for the penetration depth are obtained when approximate temperature profiles for the first time domain are introduced into the variational formulation of these problems. The solutions of such nonlinear equations are often difficult to obtain except in simple cases such as a sudden change in boundary temperature or in boundary heat flux.

Arpaci et al. [2] eliminated the use of the penetration depth by considering the variational formulation of the Laplace-transformed of unsteady diffusion problem. The method has the advantage of the absence of odd-ordered derivatives in the transformed problem, which simplifies the variational formulation. Another advantage is that the profiles that are convenient for solving the transformed problem generally yield a simple transformed solution that does not require the use of the inversion integral. He showed that the variational formulation of the Laplace-transformed diffusion problem of heat in rigid media is obtained by procedure analogous to that used for problems of rigid and deformable body mechanics.

☆ Communicated by W.J. Minkowycz.

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doi:10.1016/j.icheatmasstransfer.2007.06.010

Nomenclature

$\overline{a}_i$	Transformed parameter function (transformed generalized coordinate)
c	Speed of propagation
c_p	Specific heat
$\overline{D}$	Transformed dissipation function
g	Dimensionless internal energy generation
k	Thermal conductivity
L	Plate thickness
$\overrightarrow{n}$	Unit normal vector
P	Laplace transform parameter
$\overline{Q}_i$	Transformed thermal forces
$\overrightarrow{r}$	Position vector
T	Temperature
T_o	Initial temperature
t	Time
u'''	Internal energy generation per unit volume
$\overline{V}$	Transformed thermal potential
x, y	Cartesian coordinates

Symbols

α	Thermal diffusivity
δ	Variation symbol
η	Dimensionless distance, y/L
θ	Temperature excess, $T\text{-}T_o$
v	Integration domain
ξ	Dimensionless distance, x/L
ρ	Mass density
σ	Surface
τ	Dimensionless time
τ_r	Relaxation time
τ_r'	Dimensionless relaxation time

In the classical theory of diffusion, Fourier law of heat conduction is used to describe the relation between the heat flux vector and the temperature gradient and assumes that heat propagation speeds are infinite. When the heat transfer situations include extremely high temperature gradients, extremely large heat fluxes, or extremely short transient duration, the heat propagation speeds are finite, and the mode of conduction of heat is propagative and non diffusive. In such situations, Fourier's law includes some defects that lead to paradoxical results [3], and should be modified. The modified models include the thermal wave model, the phase-lag concept, the dual-phase-lag, and modified parabolic thermal wave equation. The improvement of Fourier's law through the concept of heat transmission by waves has been introduced by Cattaneo and Vernotte (1958) [3]. The governing equation has hyperbolic nature, with relaxation time, and is nominated as non-Fourier heat conduction equation. The non-Fourier effect becomes more and more attractive in practical engineering problems such as the non-homogenous-solid-conduction process, the rapid heating process, and the slow-conduction process. The lasers are widely used as a welding, cutting, surface treatment, surface cleaning, or heating biological tissues tool. The lasers can be considered as continuous or pulsed, stationary or moving, and point, line, or surface heat sources.

During the past few decades, many non-Fourier heat-conduction problems have been investigated. Lor and Chu [4] analyzed the problem with the interface thermal resistance. Antaki [5] discussed heat transfer in solid-phase reactions. Sanderson et al. [6] and Liu et al. [7] investigated laser-generated ultrasound models. Lin [8] computed the non-Fourier fin problems under periodic thermal conditions. However, it is very difficult to apply analytical schemes to investigate problems with complicated geometries or variable thermal properties. Due to this difficulty, attention to numerical