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The boundary element method applied to forced convection heat problems ☆

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Abstract

An integral equation formulation for steady flow of a viscous fluid is presented based on the boundary element method. The continuity, Navier–Stokes and energy equations are used for calculation of the flow and temperature fields. The governing differential equations, in terms of primitive variables, are derived using velocity–pressure–temperature parameters. The calculation of fundamental solutions and solutions tensor is shown. Applications to simple flow cases, such as driven cavity, forward facing step, deep cavity and channel are presented. Convergence difficulties are indicated, which have limited the applications to flows of low Reynolds numbers.

Keywords: Boundary element; Fundamental solution; Integral equation; Forced convection

1. Introduction

The need for solution of the system of partial differential equations, which model the flow of a fluid in channels such as pipes, blade passages, nozzles and others, appeared the very first day the fluid flow was modeled. The difficulties involved in obtaining closed solutions, even for very simple flows, required the development of clever techniques, but only with the application of numerical solutions to that system of equations, some flows of practical interest were calculated.

Despite in the period from 1970 to 1987 about 2700 references have been published, the development of the BEM did not happen quickly as the finite element method, which in the same period was contemplated with more than 23,400 publications (Mackerle[1]). The method used for fluid flow calculation is still in its beginning of development. In this work, the BEM is implemented for application to problems of fluid and heat transfer. The fundamental solution and the fundamental tensors are developed in detail. Katz [2] explains the difficulties with the implementation of the BEM, usually resulting in excessive time for implementation and validation when compared with the finite differences and the finite volumes counterparts. The work of Katz may give important hints for the BEM implementation.

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Potential flows, viscous flows modeled with the Navier–Stokes equations, heat conduction, natural and forced convection, radiation, non-Newtonian flows, Navier–Stokes equations in terms of vorticity to name just a few problems, can be calculated using the BEM.

For the Navier–Stokes equations, it is difficult to generate a fundamental solution that takes account of the non-linear terms associated with the convective derivatives. It is also required the construction of grids over the domain.

The boundary element method, nevertheless, has progressed differently depending on the areas where it has been applied: fastest in areas related to solid mechanics and acoustics problems, (Brebbia and Walker [3]; Brebbia, Telles and Wrobel [4]; Banerjee and Butterfield [5]) and slowest in the fluid mechanics.

A didactical approach is used in this work. The method of the boundary elements is applied to fluid problems, aiming also at introducing the methodology to new users. The computational implementation is based on the Kakuda and Tosaka [6] reports. There, the boundary element method uses a reformulation of the unsteady Navier Stokes equations in terms of velocity components only, by making use of the penalty function method, an approach successfully applied to flow analysis with finite element. The effectiveness of this method was illustrated by several numerical examples. Tosaka and Onishi [7,8] proposed new integral representations for the Navier Stokes equations for both steady and unsteady flow problems. The workability and validity of the methodology developed therein were shown with several numerical results for steady problems (Tosaka, Kakuda and Onishi [9]; Tosaka and Kakuda [10]; Tosaka [11]).

Aydin and Fenner [12] can also be referenced as authors of researches in the areas of fluid flow and heat transfer aiming at flow calculation using other techniques for the integration of the differential equations, including the effects of laminar viscosity. A direct iterative scheme is used to cope with the non-linear character of the integral equations. To achieve convergence, an under-relaxation technique is employed at relatively high Reynolds numbers. Numerical examples of Poisseuille and backward-facing step flow problems are considered. It is found that BEM gives accurate solutions up to a certain Reynolds number for each case. Zuo and Faghri [13] developed a model to solve the transient flow in heat pipes using a quasi-steady-state one-dimensional vapor flow and a transient two-dimensional wall and wick heat conduction. The assumption of quasi-steady-state vapor was because the vapor dynamics have a much smaller timescale than the wall conduction. A boundary element method was proposed to solve the transient two-dimensional heat conduction problem. The vapor flow dynamics was solved by a numerical approach, including two iterative “estimate-correction” processes. The model and the solution methods were verified against previous experimental and numerical studies. This work provides an effective means of transient heat pipe analysis. In the work of Young et al. [14], the boundary element technique is used to solve the steady-state convection-diffusion problems with constant velocity in a two-dimensional domain with a free interface. These problems arise in a number of important heat transfer applications involving melting or solidification, such as bulk crystal growth in Bridgman furnaces. The boundary element approach reduces the dimension of the problem, thereby improving the computational efficiency, and is particularly well suited to free-surface problems in which the position and shape of the solid–liquid interface are of primary importance. Results are presented for a case study problem representing solidification in a two-dimensional, rectangular configuration.

Although integral methods were available several decades ago, for the application to flow problems of practical interest, a comprehensive study of the formulation and application to flow problems are still being considered more recently, as they are expected to alleviate sensibly the storage and hopefully CPU time. An apparent advantage is less computational effort since volume integrals are transformed into surface integrals. But, some disadvantages arise: higher mathematical complexity to produce an usable computational formulation; need for the calculation of singular integrals; dense matrices whose inversion is more time consuming if compared to the banded matrices in the finite difference and in the finite element schemes.

Application of the boundary element method to the following fluid problems are shown: a) box with moving lid, b) stepped channel, c) deep cavity flow with heat transfer and d) channel flow with heat transfer.

2. Statement of the problem

The viscous non-dimensional transport equations (mass, momentum and energy) are presented for steady and incompressible flows. The methodology can be extended for compressible and transient flows (Ramirez and Lacaz-Santos [20,26]), as well as in natural convection, diffusion and other problems for Newtonian and non-Newtonian fluids.