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Transient heat flux function estimation utilizing the variable metric method ☆

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Abstract

A new method is presented to solve inverse heat conduction problems (IHCP's). The method belongs to the whole domain type of IHCP and utilizes the variable metric method (VMM) instead of the conjugate gradient method (CGM) in order to minimize the function of sum of square errors. Appropriate formulations for sensitivity coefficients, objective function and its gradient are set up in a manner suitable for computer programming of VMM. Numerically simulated data are utilized to assess the effectiveness of the method in comparison with the conjugate gradient method in estimation of space and time varying heat flux. Results indicate that the presented method is quite faster and more accurate than the conjugate gradient method in estimation of unknowns in the whole domain inverse problems.

Keywords: IHCP; VMM; CGM; Flux estimation

1. Introduction

Methods of solving inverse heat transfer problems (IHTP) in general can be separated into two categories: sequential methods and the whole domain methods. Each of these categories has its own advantages. Sequential methods can be used in real time mode and need less memory and computational time. Whole domain methods, on the other hand, are more accurate in estimation of the unknown parameters as compared to the sequential schemes since the whole domain methods use all of the temperature data simultaneously in estimation of any components of unknowns. They also provide more stability in inverse estimation and therefore can be used when the rate of data acquisition is very high (i.e., small time steps). Sequential schemes can not function properly in these situations; that is, they tend to be less stable than the whole domain schemes. A review of sequential methods, in general, can be found in the text book by Beck et al. [1] and in chapters 1 and 2 of Ref. [2].

In the whole domain category, all of the unknowns are determined simultaneously. Since the number of all unknowns is very high, the whole domain methods usually perform the minimization of error function using an

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Nomenclature

e_{RMS}	Relative root mean square error, Eq. (16)
f	Function of sum of the errors squares, Eq. (5)
K	Number of sensors
k	Conductivity coefficient, (W/m^2)
M	Total number of time steps
n	Total number of unknowns that should be estimated
n_s	Number of spatial components of the heat flux
q	Heat flux, (W/m^2)
$\vec{q}$	Unknowns vector (design vector with dimension $n\times 1$), Eq. (2)
S	Vector of descent direction, Eq. (6)
T	Temperature calculated by the model, (K)
t	Time, s
X	Pulse sensitivity coefficient (K m^2/W) defined by Eq. (3)
Y	Temperature measured by sensors
(x, y)	Cartesian coordinate system
Greek	
ε	Tolerance error
λ	Search step length
λ^*	Optimal step length
∇	Gradient
Superscript	
T	Matrix transpose symbol
n_s	Total number of spatial components of heat flux
$\tilde{n}$	Index of spatial components of flux, ($\tilde{n}=1, 2,\ldots, n_s$)
$'$	Derivative symbol
Subscript	
k	Sensor index number
M	Total number of time steps
m, $\tilde{m}$	Indices of time step, each between $1, 2,\ldots, M$
i	Iteration number of VMM or CGM cycles

iterative procedure. One of these techniques is called the conjugate gradient method (CGM) which is used widely and has been among one of the successful algorithms of IHCP. A comprehensive discussion on CGM and the whole domain methods is found in the text book by Ozicik and Orlande [3]. Many papers have been published that used CGM both in inverse conduction and in inverse convection and radiation problems. For instance, Chen et al. [4], Linhua et al. [5], Louahlia-Gualous et al. [6], Singh and Tanaka [7], Huang and Wang [8], Huang and Chen [9], Prud'homme and Jasmin [10], all used CGM in various inverse heat transfer problems.

As the efficiency is a major issue in IHCP, the quest for newer methods is an ongoing process. The aim of this paper is to employ the variable metric method (VMM) in IHCP instead of CGM. The variable metric method is a powerful technique in nonlinear optimization problems. Since IHCP can be viewed as an optimization problem, VMM is adapted here to solve whole domain IHCP's. At first, a general inverse heat conduction problem will be defined in which time varying and spatial distribution of the imposed heat flux to the surface of a body is unknown. Surface heat flux will be discretized both in the space and time and the function of sum of the squares of errors is accordingly defined. Subsequently, the variable metric method will be described in conjunction to the solution of IHCP. Finally, the results obtained by the VMM are compared with those of CGM.