Available online at www.sciencedirect.com
SCIENCE DIRECT®
International Communications in Heat and Mass Transfer 33 (2006) 401 – 410
International Communications in HEAT and MASS TRANSFER
www.elsevier.com/locate/ichmt

# Study of shear thinning fluid flow through highly permeable porous media ☆

M. Parvazinia, V. Nassehi *

*Chemical Engineering Department, Loughborough University, Loughborough, Leicestershire, LE11 3TU, UK*

Available online 17 January 2006

**Abstract**

The Brinkman equation is used to model the flow of power-law fluids in a highly permeable porous medium. Isothermal flow of shear thinning fluids in a porous medium between two impermeable parallel walls at different Darcy parameters (Da) and power-law index is studied. Both finite element method and analytical solutions are applied to solve the Brinkman equation. The analytical solution is based on a trial and error procedure. This solution reveals a channelling in the flow regime within the thin near walls boundary layer. Finite element solution is, in general, unstable but can be stabilised for a limited rang of Darcy parameter and power-law index.
© 2005 Elsevier Ltd. All rights reserved.

*Keywords:* Porous medium; Brinkman equation; Analytical solution; Finite element; Shear thinning fluids

## 1. Introduction

Fluid flow in saturated porous media with high permeability manifests some of the characteristics of the flow in the absence of a rigid matrix. Therefore in such a flow regime the inertia and fluid shear stress effects, not included in the Darcy model, may become significant affecting the flow and heat transfer characteristics [1,2]. In addition, Darcy's law is incompatible with the imposition of a no-slip condition on a solid boundary wall [3]. However, in most cases no-slip boundary conditions should be imposed both at the enclosure walls and at the interface between sections of heterogeneous layered porous media. Imposition of a no-slip boundary condition has significant effects on the streamwise velocity component near a wall [4]. Vafai and Tien [3] proposed a general model as an alternative to the Darcy equation including both inertia and fluid shear stress terms as well as convective term. Many works has been done on the flow of Newtonian fluids through porous media considering both fluid shear stress or inertia terms [1–9]. In these investigations both numerical or analytical solutions has been used. For Non-Newtonian fluids the results can only be represented by a non-linear differential equation which does not have a straightforward solution.

To model the flow in highly permeable porous domains, where the fluid itself carries some of the imposed stress, the Brinkman equation is used [10,11]. In the present work the behaviour of shear thinning fluids in flow regimes corresponding to the Brinkman model at different Darcy parameters is investigated. Application of the finite element

☆ Communicated by J.W. Rose and A. Briggs.

* Corresponding author.
*E-mail address:* v.nassehi@lboro.ac.uk (V. Nassehi).

0735-1933/$ - see front matter © 2005 Elsevier Ltd. All rights reserved.
doi:10.1016/j.icheatmasstransfer.2005.12.004

method to the solution of Brinkman flow of power law fluids generally yields unstable solutions. However, as is shown in the present study using a smoothing technique based on setting a larger tolerance factor (i.e., the error norm indicating the error norm the difference between consecutive iterations) stable and correct solution my be obtained. However, stability is not guaranteed for all values of permeability and power-law index. Both numerical and analytical solutions show that flow channelling takes place in the boundary layer.

## 2. Governing equations

The governing equations of the present model are derived assuming that the solid matrix and fluid passing through it form a continuum [12]. After volume averaging, these equations are written as:

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0. \tag{1}$$

Momentum equation

$$\begin{cases} -\dfrac{\partial P}{\partial x}-\dfrac{\eta}{k}u+\dfrac{\partial}{\partial x}\left(2\eta\dfrac{\partial u}{\partial x}\right)+\dfrac{\partial}{\partial y}\left[\eta\left(\dfrac{\partial v}{\partial x}+\dfrac{\partial u}{\partial y}\right)\right]=0 \\ -\dfrac{\partial P}{\partial y}-\dfrac{\eta}{k}v+\dfrac{\partial}{\partial x}\left[\eta\left(\dfrac{\partial u}{\partial y}+\dfrac{\partial v}{\partial x}\right)\right]+\dfrac{\partial}{\partial y}\left(2\eta\dfrac{\partial v}{\partial y}\right)=0 \end{cases}. \tag{2}$$

In which $k$ is permeability of the porous medium. It is assumed that the media is homogeneous, isotropic and $\eta=\eta_{\mathrm{f}}=\eta_{\mathrm{e}}$. Where $\eta_{\mathrm{e}}$ is the effective viscosity and $\eta_{\mathrm{f}}$ is the fluid viscosity. For the viscosity it is supposed that the standard definition for power-law fluids remains valid for the continuum.

$$\eta=\eta_{\circ}(\mathring{\gamma})^{m-1} \tag{3}$$

where $\mathring{\gamma}$, shear rate, is given by

$$\mathring{\gamma}=\left[2\left(\frac{\partial u}{\partial x}\right)^2+2\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial u}{\partial y}+\frac{pv}{\partial x}\right)^2\right]^{1/2}. \tag{4}$$

## 3. Dimensionless set of the governing equations

To preserve the consistency of the numerical solutions we use the following dimensionless forms of the governing equations [9]

$$\begin{aligned} & y^*=\frac{y}{h}, x^*=\frac{x}{h}, u^*=\frac{u\eta_0}{\rho g h^2}, v^*=\frac{v\eta_0}{\rho g h^2}, P^*=\frac{P}{\rho g h} \\ & \tau^*_{xx}=\frac{\tau_{xx}}{\rho g h}, \tau^*_{yy}=\frac{\tau_{yy}}{\rho g h}, \tau^*_{yx}=\frac{\tau_{yx}}{\rho g h}, \tau^*_{xy}=\frac{\tau_{xy}}{\rho g h} \end{aligned} \tag{5}$$

where $\eta_{\circ}$ is consistency, $\rho$ fluid density, $g$ acceleration due to the gravity, $h$ a characteristic dimension (assumed to be equal to the gap width in a rectangular domain), $\tau_{yy}$ normal stress and $\tau_{xy}$ shear stress.

Substituting the defined dimensionless variables in Eq. (5) to Eqs. (1)–(4), we obtain continuity equation

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0 \tag{6}$$

Download English Version:

https://daneshyari.com/en/article/654873

Download Persian Version:

https://daneshyari.com/article/654873

Daneshyari.com