International Journal of Heat and Fluid Flow 58 (2016) 40–53

Contents lists available at ScienceDirect

International Journal of Heat and Fluid Flow

journal homepage: www.elsevier.com/locate/ijheatfluidflow

On stagnation pressure increases in calorically perfect, ideal gases

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A R T I C L E I N F O

Article history:
Received 26 July 2015
Revised 1 December 2015
Accepted 23 December 2015
Available online 21 January 2016

Keywords:
Stagnation pressure
Total pressure
Increase
Transport equation
Navier–Stokes
RANS

A B S T R A C T

When stagnation pressure rises in a natural or numerically simulated flow it is frequently a cause for concern, as one *usually* expects viscosity and turbulence to cause stagnation pressure to decrease. In fact, if stagnation pressure increases, one may suspect measurement or numerical errors. However, this need not be the case, as the laws of nature do not require that stagnation pressure continually decreases. In order to help clarify matters, the objective of this work is to understand the conditions under which stagnation pressure will rise in the unsteady/steady flows of compressible, viscous, calorically perfect, ideal gases. Furthermore, at a more practical level, the goal is to understand the conditions under which stagnation pressure will increase in flows simulated with the Reynolds averaged Navier–Stokes equations and eddy-viscosity turbulence models. In order to provide an improved understanding of increases in stagnation pressure for both these scenarios, transport equations are derived that govern its behavior in the unaveraged and Reynolds averaged settings. These equations are utilized to precisely determine the relationship between changes in stagnation pressure and zeroth, first, and second derivatives of fundamental flow quantities. Furthermore, these equations are utilized to demonstrate the relationship between changes in stagnation pressure and fundamental non-dimensional quantities that govern the conductivity, viscosity, and compressibility of the flow. In addition, based on an analysis of the Reynolds averaged equation (for eddy-viscosity turbulence models), it is shown that stagnation pressure is particularly likely to experience a spurious rise at the outer edges of shear layers that are undergoing convex curvature. Thereafter, numerical experiments are performed which confirm the primary aspects of the theoretical analysis.

1. Introduction

Stagnation pressure is an important scalar quantity that can be interpreted as a Galilean variant, *approximate* local measure of the energy per unit volume of a fluid. However, despite the fundamental importance of stagnation pressure, there appear to be several misconceptions regarding the plausibility and realizability of its increase. In particular, there is the assertion that stagnation pressure cannot locally exceed the value specified at the inflow. There is also a relaxed form of this assertion that states that the stagnation pressure is conserved throughout the flow domain, and that although it may locally exceed the value specified at the inflow, it must assume a lower value elsewhere in order to enable global conservation. The authors have encountered both these assertions in discussions with engineers and practicing fluid dynamicists. However, despite their prevalence, both of these assertions are incorrect, as stagnation pressure is not conserved in a general flow, and there are no physical laws that prevent local maxima that exceed boundary specifications. The purpose of this article is to derive transport equations for stagnation pressure in general flows (flows of unsteady, viscous, compressible, calorically perfect, ideal gases) in order to help allay these misconceptions. Furthermore, it is anticipated that these transport equations will serve as a valuable tool for determining when and where stagnation pressure increases will occur, and what the probable cause of such increases will be.

There have been a number of important efforts to improve the general understanding of stagnation pressure increases in fluid flows (cf. Gaible et al., 1991; Issa, 1995; Norris, 2011; van Oudheusdan, 1996; Williams, 2002). Before proceeding further, it is important to note that many of these works frequently refer to 'total pressure' instead of 'stagnation pressure'. In general, stagnation pressure and total pressure are not the same quantity, as the latter is expected to contain contributions from the gravitational potential energy, while the former neglects these contributions. However, since body forces are frequently neglected (as they are in this article), these terms are often used interchangeably. Nevertheless,

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http://dx.doi.org/10.1016/j.ijheatfluidflow.2015.12.005

in order to avoid confusion and promote consistency, this work will proceed to utilize the term 'stagnation pressure' in place of 'total pressure' throughout, as this term is arguably more descriptive.

Now amongst the works on stagnation pressure written in the last three decades, the work of Issa (1995) for steady incompressible flows stands out. In Issa (1995), a transport equation for stagnation pressure in steady incompressible flows is obtained from the Navier–Stokes (NS) equations, and it is shown that stagnation pressure can increase locally if the viscous stress transport term exceeds the viscous dissipation term in the transport equation. Thereafter, Issa obtained analytical solutions for plane stagnation flow and Stokes flow around a sphere, and demonstrated that for each flow the stagnation pressure locally increases. These results were obtained for low Reynolds numbers ($\leq$100), as increases (or decreases) in stagnation pressure for a steady flow are inversely proportional to the Reynolds number, and are therefore less pronounced in high Reynolds number flows. It should be noted that in the work of Issa (1995) the viscosity is held constant. In Gaible et al. (1991), van Oudheusdan (1996), and Williams (2002), similar results were obtained for cases in which the viscosity is held constant. In Norris (2011), these results were expanded upon, and it was demonstrated that stagnation pressure can locally increase in a steady incompressible flow with varying viscosity, and that the variation in viscosity can result in higher stagnation pressure values than if the viscosity was held constant. Analytical solutions on planar stagnation point flows (with Reynolds numbers $\leq$1000) were presented as proof of this phenomena. In each of these flows, the viscosity was required to vary in accordance with an analytical function. These efforts were not merely an academic exercise, but were meant to model the effects of variations in eddy-viscosity on stagnation pressure. For practical problems with larger Reynolds numbers of $\mathcal{O}(10^6)$, the steady incompressible NS equations are not solved directly, and it is necessary to employ the Reynolds averaged Navier–Stokes (RANS) equations in conjunction with turbulence models that introduce spatially varying viscosity into the flow to model turbulent eddies (hence the term 'eddy-viscosity'). The net overall effect of these models is not entirely understood, although, it is immediately clear that they significantly lower the local Reynolds numbers in some regions (Norris and Richards, 2010; Richards and Norris, 2011), allowing for potential changes (increases or decreases) in stagnation pressure to become amplified (as the inverse variation with the Reynolds number no longer dampens these potential changes). Although Norris (2011) did not directly explore this effect at high Reynolds number, his results indicate that local Reynolds number variations (in the neighborhood of low Reynolds numbers) have a non-negligible effect on stagnation pressure.

The aforementioned work is significant and provides useful insights to engineers and researchers. However, there remain several unanswered questions. In particular, it is unclear precisely how stagnation pressure behaves in unsteady flows of viscous, compressible, calorically perfect, ideal gases. In fact, to the authors' knowledge, transport equations for stagnation pressure have not been derived and examined in the context of unsteady, compressible flows. In addition, an analogous set of transport equations has yet to be obtained for the RANS equations in conjunction with the associated eddy-viscosity turbulence models (EVTMs). The objective of this work is to derive these equations and provide insights into the potential causes of stagnation pressure increases in a broad range of flows. More specifically, this work attempts to provide guidelines under which fluid dynamicists can determine whether an increase in stagnation pressure is a numerical artifact or the result of an accurate solution to the governing equations, where the governing equations are the NS equations or the RANS equations with an EVTM.

The layout of this article is as follows. Section 2 presents the background necessary for constructing an unaveraged transport equation for stagnation pressure, including the governing equations for fluid flow and the associated thermodynamic relations. Section 3 presents the derivation of the unaveraged transport equation along with an analysis of this equation. Sections 4 and 5 are extensions of Sections 2 and 3 to the Reynolds averaged case (for EVTMs). Section 6 summarizes all the possible mechanisms that can cause increases in stagnation pressure. Section 7 presents the results of numerical experiments that validate the derivations and analysis from previous sections. Finally, Section 8 summarizes the results of the entire work and presents avenues for further research.

2. Preliminaries for the unaveraged case

The following discussion will focus on reviewing the conservation laws and thermodynamic relations that characterize the compressible and incompressible flows of isotropic Newtonian fluids (of which calorically perfect, ideal gases are an important type).

2.1. Conservation laws

It is useful to begin by introducing the NS equations, the conservation laws that govern a general Newtonian fluid

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_i)}{\partial x_i} = 0, \tag{1}$$

$$\frac{\partial (\rho u_i)}{\partial t} + \frac{\partial (\rho u_i u_j)}{\partial x_j} + \frac{\partial p}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j} = 0, \tag{2}$$

$$\frac{\partial \rho (e+k)}{\partial t} + \frac{\partial \rho u_i (e+k)}{\partial x_i} + \frac{\partial (u_i p)}{\partial x_i} + \frac{\partial q_i}{\partial x_i} - \frac{\partial (u_j \tau_{ij})}{\partial x_i} = 0, \tag{3}$$

where t is the temporal coordinate, x_i is the spatial coordinate in the ith dimension ($i = 1, \ldots, d$, where d is the number of spatial dimensions), ρ is the density, u_i is the flow speed in the ith direction, p is the pressure, e is the internal energy, k is the kinetic energy ($k \equiv u_i u_i/2$), τ_{ij} is the viscous stress tensor, and q_i is the heat flux. Here, the thermodynamic variables ρ and p are related to one another via an equation of state $f(p, \rho, T) = 0$, where T is the temperature. For an ideal gas the equation of state is

$$p = \rho R T, \tag{4}$$

where R is the specific gas constant.

The viscous stress tensor for an isotropic Newtonian fluid is usually modeled as follows

$$\tau_{ij} = 2\mu \left(S_{ij} - \frac{1}{3} \delta_{ij} S_{kk} \right), \tag{5}$$

where S_{ij} is the symmetric part of the velocity gradient tensor that takes the following form

$$S_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \tag{6}$$

and where μ is the shear viscosity coefficient. Note that, in accordance with standard practice, the rotational and bulk viscosity contributions to the stress tensor have been neglected (de Groot and Mazur, 2011; Heinbockel, 2001). It is also assumed that the shear viscosity coefficient's dependence on temperature (i.e., $\mu = \mu(T)$) is modeled in accordance with Sutherland's law.

Finally, the heat flux for an isotropic Newtonian fluid can be expressed in terms of the temperature gradient as follows

$$q_i = -\kappa \frac{\partial T}{\partial x_i}, \tag{7}$$

where $\kappa = \kappa(T)$ is the heat conductivity coefficient.