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An algebraic model for bypass transition in turbomachinery boundary layer flows

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ABSTRACT

A simple algebraic model is proposed for laminar to turbulent transition in boundary layers subjected to elevated free-stream turbulence. The model is combined with the newest version of the k-ω RANS turbulence model by Wilcox. The transition model takes into account, in an approximate way, two effects: filtering of high-frequency free-stream disturbances by shear and breakdown of near-wall disturbances into fine-scale turbulence. The model only uses local variables.

The model has been tuned for the flat plate T3C cases of ERCOFTAC, relevant for bypass transition and tested for flow through cascades of N3-60 (Re$=6{\cdot}10^5$) steam turbine stator vanes, V103 (Re$=1.385{\cdot}10^5$) compressor blades and T106A (Re$=1.6{\cdot}10^5$) gas turbine rotor blades. The transition model produces good results for bypass transition in attached boundary layer state and in separated boundary layer state for flows of elevated free-stream turbulence, both for 2D steady RANS and 3D time-accurate RANS simulations. Good results are also obtained for transition in separated laminar boundary layers at low free-stream turbulence by 3D time-accurate RANS simulations, thanks to resolution of the boundary layer instability and the beginning of the breakdown of the generated spanwise vortices.

1. Introduction

Transition from laminar to turbulent state in turbomachinery boundary layer flows is influenced by free-stream turbulence and pressure gradient. With a turbulence level above 0.5–1 %, the free-stream turbulence induces streamwise elongated disturbances in the near-wall region of a laminar boundary layer, termed streaks or Klebanoff distortions. They are zones of forward and backward jet-like perturbations, alternating in spanwise direction, with almost perfect periodicity. The Klebanoff disturbances grow downstream both in length and amplitude and finally break down with formation of turbulent spots. Transition is then called of bypass type, which means that the instability mechanism of the Tollmien–Schlichting waves is bypassed. Early numerical analyses of the mechanisms of bypass transition, by perturbation methods and by direct numerical simulation, were done by Jacobs and Durbin (1998, 2001), followed by many others. Overviews of research on the topic and discussions of the transition mechanisms are, e.g., given by Wang et al. (2009) and Zaki (2013). Klebanoff disturbances are formed by penetration into the boundary layer of low-frequency perturbations induced by the free-stream turbulence. The strong damping of high-frequency components is called shear sheltering. The low-speed streaks lift away from the wall, causing local inflection of the streamwise velocity, both in wall-normal and spanwise directions. Consequently, low-speed streaks are intrinsically unstable by inviscid Kelvin–Helmholtz-like instability. The breakdown of the low-speed streaks into small-scale structures leading to wall turbulence is initiated by the free-stream turbulence. Due to the inherent instability of the streaks, the breakdown into turbulence is much faster than with natural transition.

In a boundary layer with laminar separation and low free-stream turbulence, transition is initiated by inviscid Kelvin–Helmholtz instability of the laminar free shear layer, with formation of spanwise vortices. They group at selective streamwise wavelengths, analogous to Tollmien–Schlichting waves in an attached boundary layer (see e.g. McAuliffe and Yaras, 2010). The roll-up vortices break down as they travel downstream. The breakdown process is rather slow with low free-stream turbulence, but, under high free-stream turbulence, the process of bypass transition with formation of streaks in the pre-transitional attached boundary layer can co-exist with the Kelvin–Helmholtz generated spanwise vortices in the separated layer. The breakdown of the vortex rolls is then strongly accelerated by perturbations due to the Klebanoff modes. For sufficiently strong free-stream turbulence, the Kelvin–Helmholtz instability may even be bypassed by the

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Nomenclature

C	blade chord (m)
C_x	axial blade chord (m)
f_{SS}	shear sheltering factor
H_{12}	shape factor: δ^*/θ
k	turbulent kinetic energy (m^2/s^2)
k_l	large-scale turbulent kinetic energy (m^2/s^2)
k_s	small-scale turbulent kinetic energy (m^2/s^2)
l_t	turbulent length scale: $\sqrt{k}/(\beta^*\omega)$ (m)
L	length of the plate (m)
u'_l	large-scale fluctuating velocity: $\sqrt{2k_l/3}$ (m/s)
u'_s	small-scale fluctuating velocity: $\sqrt{2k_s/3}$ (m/s)
Re	Reynolds number: UL/ν, UC/ν, U_eC/ν
Re_θ	momentum-thickness Reynolds number: $U_\delta\theta/\nu$
S	magnitude of shear rate tensor: $\sqrt{2S_{ij}S_{ij}}$ (s^{-1}), streamwise distance (m)
S_0	length of the suction side of the blade (m)
S_{ij}	components of shear rate tensor $S_{ij} = 1/2(\partial U_i/\partial x_j + \partial U_j/\partial x_i) - 1/3(\partial U_k/\partial x_k)\delta_{ij}$ (s^{-1})
Tu	turbulence intensity: $\sqrt{2k/3}/U_\delta$, $\sqrt{2k/3}/U_\infty$, $\sqrt{2k/3}/U$
u_τ	friction velocity $\sqrt{\tau_w/\rho}$ (m/s)
U	velocity magnitude at cascade/channel inlet (m/s)
U_e	velocity at cascade outlet (m/s)
U_δ	velocity at edge of boundary layer (m/s)
U_∞	local free-stream velocity (m/s)
U^+	dimensionless velocity: U/u_τ
y	distance to the wall (m)
y^+	dimensionless distance to the wall: $y\, u_\tau/\nu$
γ	starting function or intermittency factor
δ	thickness of the boundary layer (m)
δ^*	displacement thickness (m)
θ	momentum thickness (m)
ν	kinematic viscosity of fluid (m^2/s)
ν_l	large-scale turbulent viscosity (m^2/s)
ν_s	small-scale turbulent viscosity (m^2/s)
ν_T	total turbulent viscosity: $\nu_s+\nu_l$ (m^2/s)
ρ	density of the fluid (kg/m^3)
τ_w	wall shear stress (N/m^2)
ω	specific dissipation rate (s^{-1})
Ω	magnitude of rotation rate tensor: $\sqrt{2\Omega_{ij}\Omega_{ij}}$ (s^{-1})
CFL	Courant–Friedrichs–Lewy number
DNS	direct numerical simulation
LES	large eddy simulation
RANS	Reynolds averaged Navier–Stokes
URANS	time-accurate (unsteady) Reynolds averaged Navier–Stokes

breakdown of the streaks. So, a bypass mechanism is possible, similar as in an attached boundary layer.

The proposed transition model is formally an intermittency model. By intermittency, denoted by γ, in a position in a transitional flow is meant the fraction of time that the flow is turbulent. Laminar flow corresponds to $\gamma = 0$, while turbulent flow corresponds to $\gamma = 1$. In most intermittency transition models, intermittency is a factor multiplying the production term of the turbulent kinetic energy in a two-equation turbulence model, and the intermittency factor is derived from one or two convection–diffusion-source equations, as e.g., in the models by Langtry and Menter (2009) and Lodefier and Dick (2006), Kubacki et al. (2009) and Durbin (2012). The transition model proposed here is algebraic, however. This means that the intermittency is derived from an algebraic formula. The model relies on two basic concepts which are borrowed from the laminar fluctuation kinetic energy models of Walters and Leylek (2004) and Walters and Cokljat (2008). Laminar fluctuation kinetic energy models form a second class of contemporary transition models, which rely on a convection-diffusion-source equation for the kinetic energy of the laminar fluctuations in a pre-transitional boundary layer. Transition is started by transfer of kinetic energy of the laminar fluctuations to the turbulent fluctuations, described by a two-equation turbulence model. In the models of Walters and Leylek and Walters and Cokljat, turbulent kinetic energy in the outer part of a pre-transitional boundary layer subjected to free-stream turbulence is damped by a function expressing the shear-sheltering phenomenon. With this function, turbulence is split into a large-scale part feeding the equation for kinetic energy of the laminar fluctuations and a small-scale part feeding the equation for kinetic energy of the turbulent fluctuations. We employ the splitting concept, but not the dynamic equation for laminar fluctuation kinetic energy. The second ingredient that we borrow is the function for transition onset which activates the transfer term for fluctuation kinetic energy. We use a similar function, but directly for activation of the intermittency factor.

A preliminary version of the transition model has been described in a recent conference paper (Kubacki et al., 2015). The current version maintains the formulation of the shear-sheltering function and the starting function (intermittency factor). But, the input parameter of the starting function is changed, as well as the stress limiting of the small-scale and large-scale turbulent fluctuations. These changes are the result of testing on more cases than reported in the conference paper. The model only describes bypass transition, thus transition under a sufficiently high level of free-stream turbulence. It is tuned for bypass transition in attached boundary layers, but it functions spontaneously very well for bypass transition in separated boundary layers. Flows with bypass transition can be simulated by 2D steady RANS. In the present version, there is no ingredient for modeling transition in a separated boundary layer due to inviscid Kelvin–Helmholtz instability under low free-stream turbulence level. This type of transition, however, can be resolved by 3D time-accurate RANS simulations, provided that vortex rolls are not artificially damped by false numerical turbulence. The transition model is used in such cases to shield the laminar and separated parts of the boundary layers from free-stream disturbances, allowing in this way the resolution of the breakdown of the vortex rolls.

2. Transition model formulation

2.1. Transport equations of turbulent quantities

The algebraic transition model is combined with the newest version of the k-ω turbulence model by Wilcox (2008). The equations have been implemented in the FLUENT commercial CFD package with the user defined function (UDF) subroutines. This way, the grid generation and numerical discretization methods from the package are used, but we determine ourselves the exact form of the turbulence model equations. The transport equations for turbulent kinetic energy and specific dissipation rate are

$$\frac{Dk}{Dt} = \gamma\, \nu_s S^2 - \beta^* k\omega + \frac{\partial}{\partial x_j}\left[\left(\nu + \sigma^*\frac{k}{\omega}\right)\frac{\partial k}{\partial x_j}\right], \tag{1}$$

$$\frac{D\omega}{Dt} = \alpha\frac{\omega}{k}\nu_s S^2 - \beta\omega^2 + \frac{\partial}{\partial x_j}\left[\left(\nu + \sigma\frac{k}{\omega}\right)\frac{\partial \omega}{\partial x_j}\right] + \frac{\sigma_d}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}. \tag{2}$$