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Numerical simulation of heat transfer in a pipe with non-homogeneous thermal boundary conditions

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ABSTRACT

Direct numerical simulations of heat transfer in a fully-developed turbulent pipe flow with circumferentially-varying thermal boundary conditions are reported. Three cases have been considered for friction Reynolds number in the range 180–360 and Prandtl number in the range 0.7–4. The temperature statistics under these heating conditions are characterized. Eddy diffusivities and turbulent Prandtl numbers for radial and circumferential directions are evaluated and compared to the values predicted by simple models. It is found that the usual assumptions made in these models provide reasonable predictions far from the wall and that corrections to the models are needed near the wall.

1. Introduction

Prediction of turbulent flows characterized by large temperature gradients and high heat-transfer rates is of great importance in engineering. Heat exchangers, combustion chambers, nuclear reactors and cooling systems in electronic devices are just some of the well-known examples in which significant temperature variations typically occur within the flow. In particular, the motivation for this study is the flow in the tubes of the heat receiver of concentrated solar power towers (Moore et al., 2010; Kolb, 2011). The heat receivers are formed by thin-walled metal tubes, assembled into panels. Heliostats located around the tower concentrate the solar radiation onto the tubes. Since the tubes are irradiated only on their outward facing side, they are subject to highly non-uniform heat flux. The heat transfer fluid, typically a molten nitrate salt, flows through the tubes increasing its temperature by convection. From a design point of view, the problem is complicated since the density, the viscosity and the heat conductivity of the salts are temperature dependent. Although the Reynolds numbers of operation are not extremely large, in the range $Re_b = 2U_bR/\nu = 2 \cdot 10^4$–$4 \cdot 10^4$, where U_b is the bulk velocity, R is the pipe radius and ν is the kinematic viscosity, the Prandtl numbers are large, in the range 10–20 depending on the employed salt. The operation of the plant must ensure that the temperature of the salt never reaches the decomposition temperature nor the melting temperature. This is not always easy to predict and requires a greater understanding of the temperature distribution than currently available. Such understanding may be obtained through direct numerical simulations (DNS) of fully developed turbulent flow in pipes. These simulations are becoming affordable with the recent advances in computational power, specially for the lower end of the range of Reynolds and Prandtl numbers mentioned above.

In spite of its practical relevance, turbulent heat transfer in pipes has not been so thoroughly studied through DNS as in plane channel flows. The main reason is the numerical difficulties associated with the cylindrical coordinate system and the corresponding numerical singularity along the symmetry line. There are some DNS of turbulent pipe flow without heat transfer like those of Wu and Moin (2008), El Khoury et al. (2013) and Chin et al. (2014). DNS of heat transfer in pipes with *homogeneous* heating are also available, for example Piller (2005) and Redjem-Saad et al. (2007). To the best of our knowledge DNS of pipe flow with *circumferentially-varying* heat flux are not available in the literature.

In this paper we report on the turbulent heat transfer in a pipe with circumferentially-varying heat flux by means of DNS of fully-developed turbulent flow. As a first step towards understanding the heat transfer characteristics of the pipes used in heat receivers, we simplify the problem by considering constant fluid properties and somewhat lower Reynolds and Prandtl numbers, as summarized in Table 1.

The main objective of the study is to generate a numerical database for RANS turbulence models benchmarking. We are particularly interested in the improvement of eddy diffusivity models,

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Table 1
Parameters of the simulations. $Re_\tau = u_\tau R/\nu, Re_b = U_b 2R/\nu$, where R is the pipe radius, u_τ is the friction velocity, U_b is the bulk velocity and ν is the kinematic viscosity.

Case	Re_τ	Re_b	Pr	Line style
1	180	$5.26 \cdot 10^3$	0.7	Solid (Black online)
2	180	$5.26 \cdot 10^3$	4	Dashed (Red online)
3	360	$1.16 \cdot 10^4$	0.7	Dashed-dotted (Blue online)

since there is a need to use very simplified models in some practical applications as the preliminary design of the heat receivers of concentrated solar power towers (Flores et al., 2014).

From experimental studies it is clear that the effective thermal diffusivity in a circular pipe is significantly non-isotropic, being higher in the circumferential than in the radial direction, as reported for example by the experiments of Quarmby and Quirk (1972). Besides this empirical evidence, many RANS calculations of the turbulent heat-transfer still use isotropic models for the thermal eddy-diffusivity as the one employed by Reynolds (1963). Gärtner et al. (1974) improved on Reynolds results by employing a non-isotropic model. Later, Launder (1978) suggested that, for an axisymmetric fully-developed velocity field, the ratio of circumferential-to-radial heat eddy diffusivities can be approximated by the ratio of the corresponding mean square velocity fluctuations. Baughn et al. (1984) applied this model to the case of a pipe with a top-half heating distribution (constant heat flux on one half of the circumference but adiabatic conditions on the other half) obtaining remarkable better results that when using an isotropic eddy diffusivity model. The DNS database reported in this paper will allow to assess the accuracy and validity of such models.

The structure of the paper is as follows. In Section 2 the computational setup is presented, including the governing equations and the boundary conditions. Results are presented in Section 3. First, the temperature statistics are characterized. This is followed by the evaluation of eddy diffusivities and turbulent Prandtl numbers. Finally, conclusions are presented in Section 4.

2. Governing equations and computational setup

As discussed in the introduction, the flow configuration studied in the present paper is a pressure-driven incompressible flow of a viscous fluid in a smooth circular pipe of radius R, subjected to a circumferentially-varying heat flux. The fluid has constant density, ρ, kinematic viscosity, ν, thermal diffusivity, α, and specific heat, C_p. Since gravity effects are not considered in the present study, the fluid temperature is simply treated as a passive scalar. Hence, the system of equations that need to be solved are the Navier–Stokes equations for an incompressible fluid (continuity and momentum), together with an advection-diffusion equation for the temperature.

In the following the nomenclature used is (x_1, x_2, x_3) for the three cartesian coordinates, with corresponding velocity components (u_1, u_2, u_3). Due to the geometry of the problem, it is convenient to define also cylindrical coordinates (r, θ, z) and velocities (u_r, u_θ, u_z), where $z = x_3$ is the axial coordinate along the pipe axis (see Fig. 1). Several averages will be used throughout the paper. The brackets $\langle\cdot\rangle$ indicate mean values, averaged in time and over the homogeneous directions. Primed variables denote fluctuations with respect to these mean values. Bulk variables, denoted with a b subindex, are averaged in time and over the cross-plane (r, θ).

The boundary conditions imposed at the wall are no-slip for the velocity and a circumferentially-varying heat flux given by

$$q_w(\theta) = \pi \overline{q}_w \sin\theta, \quad 0 < \theta < \pi \tag{1}$$

$$q_w(\theta) = 0, \quad \pi < \theta < 2\pi \tag{2}$$

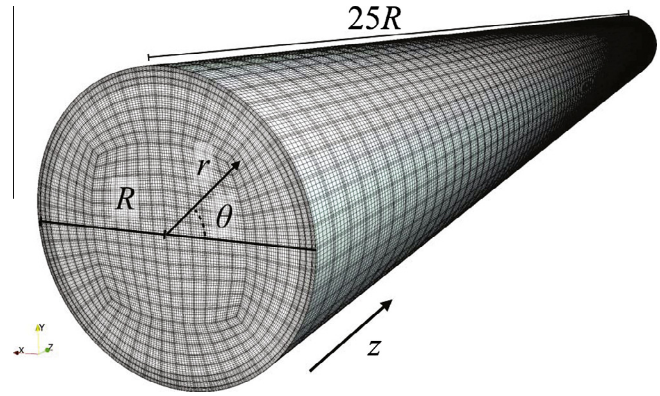

Fig. 1. Computational domain and mesh.

where $\overline{q}_w$ is the net heat flux at the wall, which corresponds to the value of the heat flux in a homogeneous heating case with the same total added heat to the system. The imposed heat flux aims to reproduce the heat transfer conditions of the pipes in a heat receiver, where the sun radiation only affects half of the circumference, while the other half can be considered adiabatic. Note that the added heat leads to a net increase of the temperature along the axial direction. A heat balance in a thin slab shows that the bulk temperature T_b increases linearly with z, with a slope given by

$$\frac{\mathrm{d}T_b}{\mathrm{d}z} = \frac{2\overline{q}_w}{\rho C_p U_b R}, \tag{3}$$

where U_b is the bulk velocity.

The net heat flux $\overline{q}_w$, together with the friction velocity, u_τ, allows the definition of a characteristic friction temperature $T^* = \overline{q}_w/\rho C_p u_\tau$. When the equations are normalized using the pipe radius R, the friction velocity u_τ and the friction temperature T^*, the only non-dimensional parameters that control the heat transfer are the Reynolds number $Re_\tau = u_\tau R/\nu$ and the Prandtl number $Pr = \nu/\alpha$. Three cases are defined with the values of Re_τ and Pr summarized in Table 1.

The linear increase of T_b with z allows us to simplify the advection–diffusion equation for the temperature, by decomposing the temperature field into $T_b(z) + T(r, \theta, z, t)$. The evolution equation for the latter is

$$\frac{\partial T}{\partial t} + u_i \frac{\partial T}{\partial x_i} = \alpha \frac{\partial^2 T}{\partial x_i \partial x_i} - u_3 \frac{\mathrm{d}T_b}{\mathrm{d}z}, \tag{4}$$

where the last term acts as a source term. Note that, since dT_b/dz is constant, the axial direction is homogeneous for $T(r, \theta, z, t)$.

Eq. (4), together with the continuity and momentum equations, are solved using the massively parallel spectral-element method (SEM) solver Nek5000. This code has been developed by Fischer et al. (2008), and it solves the incompressible Navier–Stokes equations on Gauss–Lobatto–Legendre nodes. It essentially divides the physical domain into a number of hexahedral elements where the equations of motion are solved by means of local approximations based on high-order orthogonal polynomials basis. Time is advanced with a 3rd order mixed Backward Difference/Extrapolation (BDF3/EXT3) scheme. Along with its efficient parallelization, this code provides spectral accuracy with geometrical flexibility, which makes it suitable for some engineering problems.

The size of the computational domain is selected following El Khoury et al. (2013), who performed DNS of turbulent pipe flow (without heat transfer) also with Nek5000. The computational domain is shown in Fig. 1, and it consists of a circular pipe of length $25R$. Since the z direction is homogeneous, periodic boundary