International Journal of Heat and Fluid Flow 53 (2015) 195–209

Contents lists available at ScienceDirect

International Journal of Heat and Fluid Flow

journal homepage: www.elsevier.com/locate/ijhff

Development of an explicit algebraic turbulence model for buoyant flows – Part 2: Model development and validation

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ARTICLE INFO

Article history:
Received 28 January 2014
Received in revised form 6 June 2014
Accepted 28 July 2014
Available online 24 August 2014

Keywords:
Turbulence modeling
Explicit algebraic model
Buoyancy
Mixed convection
Natural convection

ABSTRACT

A new model for buoyancy driven flows is proposed. It couples an Explicit Algebraic Reynolds Stress Model (*EARSM*) and an Explicit Algebraic Heat Flux Model (*EAHFM*) aiming to reproduce the coupled effects of flow dynamics and heat transfer via the buoyancy terms. The new model is based upon the Wallin and Johansson (2000) model for the *EARSM* and upon the Wikström, Wallin and Johansson (2000) model for the *EAHFM*. The two models are extended to account for buoyancy. Wall treatments based on the elliptic blending technique for both *EARSM* and *EAHFM* are implemented. A $k-\omega-k_\theta-r$ model supplies the turbulent scales, solving transport equations for the turbulent kinetic energy, the specific dissipation and half the thermal variance together with an algebraic equation for the turbulent time-scale ratio. The coupling is performed by an iterative process. This model allows to consider the buoyancy and wall blocking effects whatever the flow configuration. Applications of the new model on the differentially heated vertical plane channel flow lead to encouraging results whatever the convection regime.

1. Introduction

Buoyancy induces coupling between fluid dynamics and heat transfer and introduces a privileged direction in the momentum equation. These two features cannot be reproduced with the classical closure models like eddy viscosity or simple gradient hypothesis. Second order models solve transport equations for the Reynolds stresses and the turbulent heat fluxes so that they are able to deal with these effects. Nevertheless, the use of two coupled second order models is very demanding from a computational resources point of view. Algebraic approaches represent only a small additional computational effort compared to eddy viscosity models so that, as already stated by Hanjalić (2002), "*algebraic models based on a rational truncation of the differential second-moment closure are proposed as the minimum closure level for complex flows*".

Firstly developed by Girimaji and Balachandar (1998), one of these coupled algebraic models devoted to the specific case of the Rayleigh–Bénard configuration. So et al. (2004) proposed an algebraic model for general buoyant flows but only assessed it for two-dimensional homogeneous buoyant turbulent shear flow. This model is numerically difficult to implement because of a complex formulation. The model also suffers from the lack of specific wall treatment. Violeau (2009) used coupled algebraic models, simplified for two-dimensional horizontally stably stratified flow, to point out the good predictions with this approach and the significance of the turbulent scale equation. More recently, Lazeroms et al. (2013) set up an algebraic model with a complete formulation and including wall treatment. However, the model was dedicated to horizontally stably stratified flows and many constants were tuned to simplify the model for this specific case. Some other authors developed algebraic models, only for turbulent heat fluxes to be coupled with differential Reynolds stress models.

In an effort to develop a general coupling of explicit algebraic models for the Reynolds stresses and the turbulent heat fluxes for *CFD* codes, the first part (*i.e.* Vanpouille et al. (2013)) assessed the prerequisites of algebraic model development. From the analysis of *DNS* of vertical differentially heated flows, the weak equilibrium assumption was validated for Reynolds stresses and turbulent heat fluxes whatever the convection regime. The *DNS* analysis also put forward the need of an appropriate wall treatment. The pressure terms modeling was also examined in order to select the best models and validate the constants related to the buoyant contribution. The present paper carries on the development and the validation of the model which couples algebraic models for both Reynolds stresses and turbulent heat fluxes.

This paper is organized as follows. In Section 2, the full model is developed. The first step of the modeling strategy is the Explicit Algebraic Reynolds Stress Model (*EARSM*), followed by its specific wall treatment. The Explicit Algebraic Heat Flux Model (*EAHFM*) and its specific wall treatment come in second. Then, the turbulent

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http://dx.doi.org/10.1016/j.ijheatfluidflow.2014.07.006
0142-727X/

scale model is detailed. The section ends with the coupling method. Section 3 deals with assessment of the new model, using *a priori* tests and full computations on the vertical channel flow configuration.

2. Model development

As stated out by So et al. (2004) or more recently by Lazeroms et al. (2013), an algebraic model for buoyant flows is made of a Reynolds stress model, a heat flux model, transport equations for the turbulence scales and their coupling. The *DNS* analysis of the weak equilibrium assumptions pointed out the need of near-wall models for both Reynolds stresses and heat fluxes. It must be noticed that all the results are given for two-dimensional flows but are also relevant for three-dimensional flows as a first approximation.

2.1. Explicit Algebraic Reynolds Stress Model (EARSM)

Algebraic approach is based upon the weak equilibrium assumption. This assumption was validated on the differentially heated vertical plane channel flow for each convection regime and for both Reynolds stresses and turbulent heat fluxes (Vanpouille et al., 2013). The weak equilibrium assumption applied to the transport equation of the anisotropy tensor $a_{ij} = \overline{u_i'u_j'}/k - 2/3\delta_{ij}$ leads to the following equilibrium equation:

$$P_{ij} + G_{ij} + \phi_{ij} - \varepsilon_{ij} = \frac{\overline{u_i'u_j'}}{k}(P_k + G_k - \varepsilon) \quad (1)$$

where P_{ij}, G_{ij}, ϕ_{ij} and ε_{ij} are respectively the production, buoyant, redistribution and dissipation terms in the Reynolds stress transport equation and P_k, G_k and ε the production, the buoyant term and the dissipation rate in the turbulent kinetic energy k transport equation, defined as:

$$\begin{aligned}
P_{ij} &= -\overline{u_i'u_k'}\frac{\partial U_j}{\partial x_k} - \overline{u_j'u_k'}\frac{\partial U_i}{\partial x_k} \quad & P_k &= \frac{1}{2}P_{ii} \\
G_{ij} &= -\beta\left(g_i\overline{u_j'T'} + g_j\overline{u_i'T'}\right) \quad & G_k &= \frac{1}{2}G_{ii} \\
\varepsilon_{ij} &= \nu\overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}} & \varepsilon &= \frac{1}{2}\varepsilon_{ii} \\
\phi_{ij} &= \overline{\frac{p'}{\rho}\left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right)}
\end{aligned} \quad (2)$$

where is β the volumetric expansion, U_i the mean velocity, u_i' the velocity fluctuation, T the mean temperature and T' the temperature fluctuation.

2.1.1. Homogeneous model

The present model derivation is based upon Wallin and Johansson (2000). In Eq. (1), the buoyant terms G_{ij} and G_k are linked to the turbulent heat fluxes and are thus considered as inputs from the turbulent heat flux model. A general quasi-linear model for the redistribution term ϕ_{ij} is:

$$\begin{aligned}
\phi_{ij} = &-\left(c_1 + c_1^*\frac{P_k}{\varepsilon}\right)\varepsilon a_{ij} + c_2 k S_{ij} \\
&+ c_3 k\left(a_{ik}S_{kj} + S_{ik}a_{kj} - \frac{2}{3}a_{kl}S_{lk}\delta_{ij}\right) - c_4 k\left(a_{ik}\Omega_{kj} - \Omega_{ik}a_{kj}\right) \\
&- c_6\left(G_{ij} - \frac{2}{3}G_k\delta_{ij}\right)
\end{aligned} \quad (3)$$

The set of constants from So et al. (2004) (Table 1) with $c_6 = 0.6$ for buoyancy term is chosen as it offers a good compromise whatever the convection regime (Vanpouille et al., 2013). Nevertheless,

Table 1
Constants of the So et al. (2004) (*SJG*) redistribution model.

Model	c_1	c_1^*	c_2	c_2^*	c_3	c_4	c_5	c_6
SJG	1.7	0.9	0.36	0	0.625	0.2	0	0.6

the general linear form of Eq. (3) is used here for the redistribution term, so that users are free to choose any redistribution model. The dissipation model is an isotropic model:

$$\varepsilon_{ij} = \frac{2}{3}\varepsilon\delta_{ij} \quad (4)$$

The normalized strain tensor S_{ij}^* and vorticity tensor Ω_{ij}^* are defined as:

$$S_{ij}^* = \frac{\tau}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) \quad \text{and} \quad \Omega_{ij}^* = \frac{\tau}{2}\left(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i}\right) \quad (5)$$

with $\tau = k/\varepsilon$. Eq. (1) can be rearranged as:

$$\begin{aligned}
N'\underline{\underline{a}} = &-A_1\underline{\underline{S}}^* - A_2\left(\underline{\underline{a}}\cdot\underline{\underline{S}}^* + \underline{\underline{S}}^*\cdot\underline{\underline{a}} - \frac{2}{3}\,\mathrm{tr}\,\left(\underline{\underline{a}}\cdot\underline{\underline{S}}^*\right)\underline{\underline{I_d}}\right) \\
&+ A_3\left(\underline{\underline{a}}\cdot\underline{\underline{\Omega}}^* - \underline{\underline{\Omega}}^*\cdot\underline{\underline{a}}\right) + \frac{A_4}{\varepsilon}\left(\underline{\underline{G}} - \frac{2}{3}G_k\underline{\underline{I_d}}\right)
\end{aligned} \quad (6)$$

where $N' = A_5 + A_6P_k/\varepsilon + A_7G_k/\varepsilon$ and $A_1 = 4/3 - c_2$, $A_2 = 1 - c_3$, $A_3 = 1 - c_4$, $A_4 = 1 - c_6$, $A_5 = c_1 - 1$, $A_6 = c_1^* + 1$ and $A_7 = 1$ constants are linked to the redistribution model constants. The term $\left(\underline{\underline{G}} - 2/3\ G_k\underline{\underline{I_d}}\right)$ will be noted $\underline{\underline{\Gamma}}$ for conciseness.

In order to derive an explicit expression, the anisotropy tensor is projected on a tensor basis. For two-dimensional forced convection flows, the tensor basis reduces to three tensors (Pope, 1975):

$$\underline{\underline{S}}^*;\ \left(\underline{\underline{S}}^{*2} - \frac{1}{3}\mathrm{tr}\left(\underline{\underline{S}}^{*2}\right)\underline{\underline{I_d}}\right);\ \left(\underline{\underline{S}}^*\cdot\underline{\underline{\Omega}}^* - \underline{\underline{\Omega}}^*\cdot\underline{\underline{S}}^*\right) \quad (7)$$

When the velocity gradient goes to zero, this tensor basis degenerates and gives a null anisotropy tensor, *i.e.* an isotropic state. This is not satisfactory for buoyant flows as buoyancy can also induce anisotropy. For a null velocity gradient, expression (6) reduces to:

$$N'\underline{\underline{a}} = \frac{A_4}{\varepsilon}\left(\underline{\underline{G}} - \frac{2}{3}G_k\underline{\underline{I_d}}\right) \quad (8)$$

To model buoyancy effects, So et al. (2004) complement the basis with two tensors derived from $\underline{\underline{\Gamma}}$ and related to the two-dimensional and three-dimensional identity tensors. The two-dimensional identity tensor is difficult to define for three-dimensional flows. Lazeroms et al. (2013) complement the basis to finally deal with a ten tensor basis for two-dimensional flows. When the velocity gradient is not zero, these tensor bases are difficult to handle, which leads to a complex formulation to model the anisotropy induced by the buoyancy.

The proposed strategy is to isolate the solution without velocity gradient (8), which can be expressed as $\underline{\underline{a}} = \gamma_4\left(\underline{\underline{G}} - \frac{2}{3}G_k\underline{\underline{I_d}}\right)$ with $\gamma_4 = A_4/(N'\varepsilon)$ and to project the anisotropy tensor minus this solution on the original basis as:

$$\begin{aligned}
\underline{\underline{a}} - \gamma_4\left(\underline{\underline{G}} - \frac{2}{3}G_k\underline{\underline{I_d}}\right) = \gamma_1\underline{\underline{S}}^* + \gamma_2\left(\underline{\underline{S}}^{*2} - \frac{1}{3}\mathrm{tr}\left(\underline{\underline{S}}^{*2}\right)\underline{\underline{I_d}}\right) \\
+ \gamma_3\left(\underline{\underline{S}}^*\cdot\underline{\underline{\Omega}}^* - \underline{\underline{\Omega}}^*\cdot\underline{\underline{S}}^*\right)
\end{aligned} \quad (9)$$

Introducing this anisotropy tensor representation into the algebraic Eq. (6), the projection coefficients γ_i are determined as functions of N' and G_k/ε. The so-obtained projection coefficients γ_i read: