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Parametric perturbative study of the supercritical cross-flow boundary layer

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ABSTRACT

A linear analysis of the transient evolution of small perturbations in the supercritical FSC cross-flow boundary layer is presented. We used the classical method based on the temporal evolution of individual three-dimensional travelling waves subject to near-optimal initial conditions and considered an extended portion of the parameter space. Our parametrization included the wave-number, the wave-angle, the cross-flow angle, the Hartree parameter and the Reynolds number. Special focus was given to the role played by the waveangle in inducing very steep initial transient growths in waves that proved to be stable in the long term.

We found that the angular distribution of the asymptotically unstable waves and of the waves that show a transient growth depends greatly on the value of the cross flow angle and wave-angle as well as on the sign of the Hartree parameter, but depend much less on the Reynolds number. In the case of the decelerated boundary layer, at sufficiently short wavelengths, transient growths become much more rapid than the initial growth of the unstable waves. In all cases of transient growth, pressure perturbations at the wall are not synchronous with the kinetic energy of the perturbation.

We present a comparison with the sub-critical results obtained by Breuer and Kuraishi (1994) ($Re = 500$, sweep angle of $\pi/4$) for the same full range of the obliquity angle here considered (π radiants).

1. Introduction

The cross flow boundary layer is one of the most important boundary layers in engineering applications (aerospace, mechanical, wind...), cf. the recent review by Saric et al. (2003) and the monographs Schmid and Henningson (2001), Criminale et al. (2003). Examples of cross flow boundary layer include flow over a swept back air plane wing, rotating discs, cones and spheres and cones at an angle of attack. It is important to understand the dynamics of this flow and to learn how to prevent the possibility of breakdown to turbulence. Furthermore, unlike the well-known Blasius boundary layer, breakdown is far more likely in this flow. For example, it can be unstable inviscidly as well as that caused by the influence of viscosity due to the existence of an inflexion point in the mean profile (Gregory et al., 1955). This work presents a study in an extended portion of the parameter space of the stability of the cross flow boundary layer in supercritical conditions with three-dimensional perturbations based not only on the modal approach but also examining the temporal evolution of the perturbation. Flow due to an infinite rotating discs often has been used in literature as an archetypal example of three-dimensional boundary layers (Saric et al., 2003). Lingwood (1995) found that in this flow a transition from local linear convective to radial absolute instability can occur. This inspired many authors and led to the investigation of the fully non linear regime (see, among others Pier (2003), Healy (2006)).

The swept-wing boundary layer is genuinely three-dimensional, which makes its exploration very complex. Despite this complexity, Lingwood's approach motivated studies on the possibility of absolute instability operating in the swept-wing boundary layer. In particular, it was found (Lingwood, 1997) that close to the attachment line there is chordwise absolute instability above a critical spanwise Reynolds number of about 545. Taylor and Peake (1998) extended the study by Lingwood and searched for pinch points in the cross flow direction for a larger range of flow angles and pressure gradients. Although these crossflow-induced pinch points do not constitute an absolute instability, as there is no concomitant pinch occurring in the streamwise wavenumber plane, they can be used to find the maximum local growth rate contained in a wavepacket travelling in any given direction. Recently, these findings were confirmed by Koch (2002) in a work dedicated to the study of the secondary instability of stationary cross-flow vortices. In general,

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a rigorous proof that the absolute instability cannot occur in a swept-wing boundary layer does not yet exist.

The three-dimensional boundary layer has been also investigated in the context of receptivity and transient optimal perturbations. Most studies of optimal disturbances in wall-bounded flows (Luchini and Bottaro, 1998; Luchini, 2000) deal with temporal growth of perturbations. For example, Corbett and Bottaro (2001) performed a local stability analysis using a variational technique in the temporal framework. They found that the three-dimensional boundary layer shows significantly greater capacity for algebraic growth than the two-dimensional boundary layer with the same base flow parameters. Moreover, they proved that the cross flow angle that maximizes the transient growth is nearly equal to 49°. Schrader et al. (2009) and Tempelmann et al. (2010) studied the receptivity problem for spatial growing perturbation considering vortical free stream modes, free stream turbulence and surface roughness. They found that steady cross-flow instabilities to dominate for low-level free stream disturbance. Malik et al. (1994, 1999) investigate the secondary instability characteristics of swept-wing boundary and found that three types of secondary disturbances can be distinguished. The first two were high-frequency disturbances with high growth rates and maxima located away from the wall. Their origin was related to regions of high spanwise shear (type I) and vertical shear (type II). The third type is a low-frequency disturbance with smaller growth rates and maxima closer to the wall representing a primary travelling crossflow disturbances being modulated by the stationary crossflow vortex.

This work treats the linear perturbation problem and demonstrates the importance of the results during the transient period as well the long time behaviour. Near-optimal perturbations which are localized within the boundary layer thickness are used as initial conditions (Lasseigne et al., 1999; Corbett and Bottaro, 2001). We also have good agreement with results obtained by using impulsive forcing (Taylor and Peake, 1998) or least-damped Orr–Sommerfeld eigenfunctions as initial conditions (Breuer and Kuraishi, 1994). The extreme simplicity of this method allows for an extended study of the parameter space. In particular, special attention was given to the role played by the direction of the perturbation both in the transient and in the asymptotic regime. In sub-critical conditions, a similar analysis was performed by Breuer and Kuraishi (1994). They observed that, when the external flow is accelerated, the disturbances which have greater transient growth are those that propagate in the crossflow direction. Vice versa, if the external flow is decelerated, the maximum transient growth is obtained with disturbances propagating in the opposite cross-flow direction.

With this paper we wish to extend the study of Breuer and Kuraishi by considering supercritical conditions. The pressure perturbation during the transient is also investigated and in particular is investigated when the maximum amplification factor for the pressure measured at the wall come in advance or in delay with respect to the maximum amplification of the energy.

This paper is organized as follow. The physical problem is described in Section 2. Section 2.1 is dedicated to the mean three-dimensional flow, Section 2.2 to the definition of the initial value problem and modal analysis. Sections 3 and 4 present transient dynamics and the role of the perturbation inclination and the long term behaviour, respectively. Section 5 gives information on the wall pressure transient. Conclusions follow in Section 6.

2. Problem formulation

2.1. Mean flow

As customary, we use the parallel flow approximation to describe the linear evolution of small amplitude disturbances. When the parallel flow assumption holds, the base flow components only vary with the wall normal coordinate. The assumption behind this approach is that the mean boundary layer flow quantities vary slowly in the streamwise direction compared to the disturbance quantities. In general, to account for nonparallel effects in diverging flows, the spatial formulation of the governing perturbative equations is used, see for example the multiple scale analysis carried out by El-Hady (1991) who considers the nonparallel effects for subsonic and supersonic boundary layers. A specific application to the base flow analysed in this paper (the Falkner–Skan–Cooke boundary layer, with a displacement thickness Reynolds number of 490) can be found in Högberg and Henningson (1998) where, by means of linear local eigenvalue calculations compared to spatial direct numerical simulations, it is showed that nonparallel effects induce a raise in the growth rate of the order of the 13% along the streamwise direction.

In this paper, nonparallels effect are disregarded. We thus assume that locally we can represent the boundary layer as a parallel shear flow subject to small pertubations in the form of travelling waves and define two local coordinate systems as shown in Fig. 1(a) and (b). On an infinite swept wing, taken any point x^* lying on the wing, we can always distinguish the chordwise direction, x_c, from the streamline direction, x. We use the coordinate system based on the streamline direction. The y direction is normal to the wall and the z direction is normal to both x and y directions. A good approximation of the velocity profiles in a three dimensional boundary layer is given by the family of similarity solutions known as Falkner–Skan–Cooke (FSC) solutions (Cooke, 1950; Rosenhead, 1963). There are two parameters in the FSC formulation that allow the magnitude of the cross flow to be varied: β, the dimensionless pressure gradient, or Hartree parameter, and θ the crossflow angle between the streamwise direction and the chordwise direction, see Fig. 1(c). The mean vertical velocity is assumed to be zero.

It should be recalled that with this approximation the external flow is accelerating as the external pressure decreases ($\beta > 0$) and one can talk of boundary layer in a favourable pressure gradient and vice versa.

Fig. 1(b) and (c) shows the velocity profiles in this reference frame. As is customary, variables are non-dimensionalized with respect to U_e, the free-stream velocity at the boundary layer edge, and with respect to the streamwise displacement thickness, $\delta^* = \int_0^\infty (1-U)dy$. The Reynolds number is then defined as $Re = U_e\delta^*/\nu$.

2.2. Initial-value problem and modal analysis

The transient as well as the long term behaviours of arbitrary three-dimensional disturbances acting on the FSC cross-flow boundary layer are investigated. We have considered the velocity vorticity formulation and have Fourier transformed the governing disturbance equations in the streamwise and spanwise directions only, using respectively the wavenumbers α and γ. This leads to generalized forms of the Orr–Sommerfeld and Squire equations:

$$\begin{aligned}
&\left(\frac{\partial}{\partial t}+i(\alpha U+\gamma W)\right)\left(\frac{\partial^2}{\partial y^2}-k^2\right)\hat{v}-i(\alpha U''+\gamma W'')\hat{v}-\frac{1}{Re}\left(\frac{\partial^2}{\partial y^2}-k^2\right)^2\hat{v}=0, \qquad 3,\\
&\left[\frac{\partial}{\partial t}+i(\alpha U+\gamma W)-\frac{1}{Re}\left(\frac{\partial^2}{\partial y^2}-k^2\right)\right]\hat{\omega}_y=i(\alpha W'-\gamma U')\hat{v},
\end{aligned} \tag{1}$$

where $k^2=\alpha^2+\gamma^2$ is the polar wavenumber, $\hat{v}$ and $\hat{\omega}_y$ are respectively the transformed perturbation vertical velocity and vorticity, U, U', U'', W, W' and W'' indicate the base flow streamwise and spanwise profiles and their derivatives in the y direction. The boundary conditions require that $\hat{v}=\hat{v}'=\hat{\omega}_y=0$ at the wall and at infinity.

On these equations we have performed both a modal analysis and an initial value problem, which hereafter will be indicated