Forensic Science International 266 (2016) 502–510

Contents lists available at ScienceDirect

Forensic Science International

journal homepage: www.elsevier.com/locate/forsciint

A Taylor-Affine Arithmetic for analyzing the calculation result uncertainty in accident reconstruction

Tiefang Zou[a,b,*], Haitao Peng[a,b], Ming Cai[c], Hequan Wu[a,b], Lin Hu[a,b]

[a] School of Automobile and Mechanical Engineering, Changsha University of Science and Technology, Changsha 410004, China
[b] Key Laboratory of Safety Design and Reliability Technology for Engineering Vehicle, Hunan Province 410004, China
[c] School of Engineering, Sun Yat-sen University, Guangzhou,Guangdong 510275, China

ARTICLE INFO

Article history:
Received 22 January 2016
Received in revised form 26 June 2016
Accepted 9 July 2016
Available online 19 July 2016

Keywords:
Accident reconstruction
Calculation result uncertainty
Affine arithmetic
Taylor-expansion
Interval

ABSTRACT

In order to analyze the uncertainty of a reconstructed result, the Interval Algorithm (IA), the Affine Arithmetic (AA) and the Modified Affine Arithmetic (MAA) were introduced firstly, and then a Taylor-Affine Arithmetic (TAA) was proposed based on the MAA and Taylor series. Steps of the TAA, especially in analyzing uncertainty of a simulation result were given. Through the preceding five numerical cases, its application was demonstrated and its feasibility was validated. Results showed that no matter other methods (The IA, AA, the Upper and Lower bound Method, the Finite Difference Method) work well or bad, the TAA work well, even under the condition that the MAA cannot work in some cases because of the division/root operation in these models. Furthermore, in order to make sure that the result obtained from the TAA can be very close to the accurate interval, a simple algorithm was proposed based on the sub-interval technique, its feasibility was validated by two other numerical cases. Finally, a vehicle–pedestrian test was given to demonstrate the application of the TAA in practice. In the vehicle–pedestrian test, the interval [35.5, 39.1] km/h of the impact velocity can be calculated according to steps of the TAA, such interval information will be more useful in accident responsibility identification than a single number. This study will provide a new alternative method for uncertainty analysis in accident reconstruction.

1. Introduction

Traces are the foundation of accident reconstruction, a reliable result can be obtained based on these accurate traces. However, due to the influence of the environment, such as the rain, the dust, and other vehicles and so on, traces will be faded away slowly, which results in uncertain traces. The most likely situation is that the interval of a specific trace, such as the braking distance of the vehicle, will be given by the police. Under such condition, in order to enhance the objectivity and reliability of a reconstructed result, these uncertain interval information should be reflected in the reconstructed result [1,2]. It is a problem of the effect of input traces uncertainty on calculation results uncertainty, and the problem can be tackled by the help of uncertainty analysis.

Many scientists had been contributed to this field and many methods had been proposed, such as the Upper and Lower bound Method (ULM) [3], the Finite Difference Method (FDM) [4], the Response Surface Methodology (RSM) [5–7] and the Interval Algorithm (IA) [5]. The IA is the simplest method in these methods, but it has the interval expansion problem; especially, if there are higher correlations between these independent variables. In order to overcome the interval expansion problem, many other techniques were introduced, such as the random sub-interval technique [5], the optimization technique [8] and the Affine Arithmetic (AA) [9,10]. In a way, the correlations between independent variables were considered in the AA; additionally, the algorithm of this method has optimization property; these properties make results obtained from the AA are better than the IA in many cases. But the AA is not a perfect method and it will introduce error also because of the approximate operation in the division and multiplication, especially, under the condition that the model is a strongly nonlinear model [10]. For this reason, a Modified Affine Arithmetic (MAA) was proposed [11], which will improve the analyzed result significantly; but it cannot work well in the division, exponentiation and root operation.

In order to make the MAA worked under all conditions, the Taylor expansion will be introduced and a method named Taylor-

* Corresponding author.
E-mail address: tiefang@163.com (T. Zou).

http://dx.doi.org/10.1016/j.forsciint.2016.07.011
0379-0738/

Affine Arithmetic (TAA) will be proposed. In this method, the accident reconstruction model will be Taylor expanded firstly, and then the upper and lower bound of the reconstructed results will be calculated by combining the MAA and the Taylor expansion.

2. Problem description

All models within accident reconstruction can be presented as:

$$R = f(X),\ X = (x_1, \ldots, x_s)^T \quad (1)$$

where X are input traces, and R are accident reconstruction results. In most cases, R represent the vehicle velocity and/or the impact position, s represents the number of input traces. If traces in an accident include uncertain interval information, the problem of analyzing the calculation result uncertainty lies in how to calculate the interval of R, according to the interval of X.

The simplest method is the IA, In the IA, the uncertain trace is recorded as $[x] = [x^L, x^U]$, where x^L and x^U are the lower and upper bound of x independently; and then the interval of R can be calculated according to the four arithmetic operations of interval numbers, which are listed in Appendix A. Because of the interval expansion problem, the calculated interval from IA exists larger error in some cases. For example, seeking the range of $y = 3 - x - x^2 + 2x^3$ in the definition domain $x \in [1,\ 3]$, the truth interval of y is [3, 45], but the interval calculated by the IA is $y = 3 - [1, 3] - [1, 3] \times [1, 3] + 2 \times [1, 3] \times [1, 3] \times [1, 3] = [0, 2] - [1, 9] + [2, 54] = [-7, 55]$, which is too conservative.

In order to overcome the interval expansion problem, the AA [12] was introduced. In the AA, an uncertain trace is recorded as

$$\hat{x} = x_0 + x_1 e_1 + x_2 e_2 + \cdots + x_t e_t \quad (2)$$

where $\hat{x}$ is the affine form of x; x_0 is the middle value of $\hat{x}$; $e_i \in [-1, 1] (i = 1, 2, \ldots t)$ is the ith noise symbol; x_i is the ith partial increment of the $\hat{x}$, which represents the uncertain amplitude of the ith noise symbol. The expression of the interval form of $\hat{x}$ can be given as

$$x = [x^L, x^U] = \left[x_0 - \sum_{i=1}^{t} |x_i|, x_0 + \sum_{i=1}^{t} |x_i|\right] \quad (3)$$

On the contrary, for an arbitrary interval trace $[x] = [x^L, x^U]$, its affine arithmetic form can be given as

$$\hat{x} = \frac{[(x^L + x^U) + (x^U - x^L)e_1]}{2} \quad (4)$$

And then the interval of R can be calculated according to the four arithmetic operations of the affine arithmetic, which is listed in Appendix B. Because of the approximation in calculation, calculation results in multiplication and division of the AA will introduce errors also. For example, seeking the range of $y = 3 - x - x^2 + 2x^3$ in the definition domain $x \in [1,\ 3]$, make $x = x_0 + x_1 e_1$, where $x_0 = (1 + 3)/2 = 2$, $x_1 = (3 - 1)/2 = 1$, and then the expression of y can be given according to Appendix B.

$$y = 3 - x_0 + 3x_1^2 x_0 + (2x_0 - 1.5)x_0^2 + (6x_0^2 x_1 - 2x_0 x_1 - x_1)e_1 + 0.5x_1^2(6x_0 - 1)e_2 + 2x_1^3 e_3$$

Finally, the upper and lower bound of y can be given

$$y^+ = 3 - x_0 + 3x_1^2 x_0 + (2x_0 - 1.5)x_0^2 + |6x_0^2 x_1 - 2x_0 x_1 - x_1| + |0.5x_1^2(6x_0 - 1)| + |2x_1^3| = 43.5 \qquad y^- = 3 - x_0 + 3x_1^2 x_0 + (2x_0 - 1.5)x_0^2 - |6x_0^2 x_1 - 2x_0 x_1 - x_1| - |0.5x_1^2(6x_0 - 1)| - |2x_1^3| = -9.5$$

Comparing with the truth interval of $y = [3,\ 45]$, the interval calculated by the AA in this example is conservative also.

In order to overcome the problem, the Modified Affine Arithmetic (MAA) was proposed. In the MAA, for an arbitrary function f, the f will be expressed as a polynomial g firstly, and then all interval parameters will be converted to their affine form, after that the polynomial g will be converted to its affine form by matrices, the upper and lower bound of the f can be given finally. Detailed information about the MAA is listed in Appendix C. From Reference [11], results obtained from the MAA will be better than results obtained from the AA, and it is very close to the truth value. For example, seeking the range of $y = 3 - x - x^2 + 2x^3$ in the definition domain $x \in [1,\ 3]$, according to Eq. (11) in Appendix C.

$$H = \begin{bmatrix} v_{00} & 0 & 0 & 0 \\ 0 & v_{11} & 0 & 0 \\ 0 & 0 & v_{22} & 0 \\ 0 & 0 & 0 & v_{33} \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

And then according to Eq. (12) in Appendix C, considering that $x_0 = 2$, $x_1 = 1$, then $w_{00} = v_{00} + v_{11}x_0 + v_{22}x_0^2 + v_{33}x_0^3 = 13$, $w_{01} = v_{11}x_1 + 2v_{22}x_0 x_1 + 3v_{33}x_0^2 x_1 = 19$ and $w_{02} = v_{22}x_1^2 + 3v_{33}x_1^2 x_0 = 11$, $w_{03} = v_{33}x_1^3 = 2$, while all other elements in the W are 0. And then according to Eqs. (13) and (14) in Appendix C, the upper and lower bound of the y can be given

$$y^U = w_{00} + w_{01} + w_{02} + w_{03} = 45$$

$$y^L = w_{00} - w_{01} + w_{02} - w_{03} = 3$$

which is the same as the truth interval of y.

Unfortunately, such a good method did not apply to the accident reconstruction field before, and it cannot work in some operations, such as the division operation, the exponentiation operation and the root operation. But there are many root and division operation in accident reconstruction, such as the simple model used for calculating the vehicle velocity according to the braking distance, $v = \sqrt{2\mu g s}$, where μ is the friction coefficient between the road and the vehicle, and s is the braking distance. Under such condition, how to analyze the interval of a result obtained from such kind of models with the MAA becomes a meaningful problem.

3. The Taylor-Affine Arithmetic (TAA)

3.1. Steps of the TAA

In order to make the MAA work in the accident reconstruction field and considering the application of Taylor series methods in interval computation [13], the accident reconstruction model will be Taylor expanded firstly, and then the upper and lower bound of the reconstructed result will be calculated by combining the Taylor expansion and the MAA, this method was named as the TAA. Steps of the TAA are:

1) Standardized treatment of the accident reconstruction model and obtaining a new model g;
2) Do Taylor expansion of the new model g at a special point, such as the middle of each uncertain traces;
3) Calculate the interval of R according to Appendix C.

For example, as for an accident reconstruction model with two independent variables

$$R = f(x, y), (x \in (x^L, x^U), y \in (y^L, y^U)) \quad (5)$$