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A scaling investigation of the laminar convective flow in a solar chimney for natural ventilation

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A B S T R A C T

The flow behavior due to natural convection of air (with a Prandtl number less than 1) inside a solar chimney with an imposed heat flux on a vertical absorber wall is investigated by a scaling analysis and a corresponding numerical simulation. Three distinct flow regimes are identified, one with a distinct thermal boundary layer and the other two without a distinct thermal boundary layer, depending on the Rayleigh number. The two regimes without a distinct thermal boundary layer are further classified into low and medium Rayleigh number sub-regimes respectively. These sub-regimes are characterized by conduction dominance in which the thermal boundary layer grows to encompass the entire width of the channel before convection becomes important. Flow development in each of these flow regimes and sub-regimes is characterized through transient scaling, and scaling correlations are developed to describe the temperature, flow velocity and mass flow rate, which characterize the ventilation performance of the solar chimney. The scaling arguments are validated by the corresponding numerical data.

1. Introduction

Scale analysis, or an order-of-magnitude analysis, is an effective tool for revealing the underlying principles of many physical phenomena including fluid flow and heat transfer. A classic example of the scale analysis is the reduction of full Navier–Stokes equations to the boundary layer equations, which are the basis of many investigations of convective flows. Over the past three decades, scale analysis has been applied to study numerous convective flows in a variety of flow configurations involving different geometries and thermal forcing conditions.

Natural convection in an enclosure or adjacent to an isolated surface (vertical or inclined) has attracted strong research attention. Patterson and Imberger (1980) used a rectangular cavity model to carry out a transient scale analysis of the case with an instantaneous heating and cooling on two opposing vertical sidewalls. They devised various transient flow scenarios which are determined by the Rayleigh number, the Prandtl number and the aspect ratio of the cavity. Poulikakos and Bejan (1983) carried out a transient scale analysis of natural convection in a triangular enclosure filled with air with imposed isothermal conditions on the horizontal bottom and sloping roof to simulate the day- and night-time convection in an attic space. For the day time convection, the sloping roof was considered to be warmer than the horizontal bottom and for the night time convection the opposite configuration was considered. Natural convection in a triangular domain was also considered by Lei and Patterson (2002) to study the unsteady exchange flow in reservoir sidearms induced by absorption of radiation. They characterized the convective flow into different flow regimes depending on the Rayleigh number, the Prandtl number and the slope of the bottom. Lin and Armfield (2005a) have considered an enclosed cylindrical geometry to study unsteady natural convection cooling of an initially quiescent isothermal fluid with $Pr < 1$ when the vertical walls are subjected to a lower temperature. Scaling relations are established for three distinct flow stages namely a boundary layer development stage, a stratification stage and a cooling down stage. In a separate study, Lin and Armfield (2005b) carried out a transient scale analysis for an isolated vertical plate subjected to an isoflux heating for $Pr < 1$ fluid. Subsequently, Lin et al. (2009) extended the analysis to the case with isothermal heating for a $Pr > 1$ fluid and showed the existence of a three-layer structure: an inner viscous layer, a thermal boundary layer and an outer viscous layer. Armfield et al. (2007) reported a transient scale analysis of the natural convection boundary layer adjacent to an evenly heated semi-infinite vertical plate in both stratified and non-stratified ambient of a $Pr > 1$ fluid. Natural convective boundary layer adjacent to an inclined plate subjected to sudden and ramp heating condition was considered by Saha et al. (2010).

Despite that the scale analysis approach has been extensively applied to study natural convective flow in various configurations, the application of this powerful technique to an open vertical channel configuration has not been reported. This configuration can be

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Nomenclature

A	aspect ratio
A_c	cross-sectional area of a narrow channel (m^2)
Bo	Boussinesq number ($Bo = RaPr$)
c_p	specific heat (J/kg K)
D_h	hydraulic diameter (m) (=$2W_g$)
f	friction factor
g	gravitational acceleration (m/s^2)
H_a	absorber height (m)
k	thermal conductivity (W/m K)
L	distance between the two vertical walls of a narrow channel (m)
$\dot{m}$	steady state mass flow rate (kg/s m)
$\dot{M}$	dimensionless steady state mass flow rate
p	local fluid pressure (Pa)
P	dimensionless fluid pressure
Pr	Prandtl number
q''	input heat flux (W/m^2)
$\dot{Q}$	volumetric flow rate (m^3/s)
Ra	Rayleigh number
Ra_c	critical Rayleigh number
t	time (s)
t_s	steady state time for distinct thermal boundary layer (s)
t_{f1}, t_{f2}	steady state time for medium and low Rayleigh number regime (s)
t_w	diffusion time (K)
T	local fluid temperature (K)
T_0	reference fluid temperature (K)
T_b	temperature growth scale in the thermal boundary layer (K)
T_{bs}	steady state temperature scale for distinct thermal boundary layer (K)
T_{b1}, T_{b2}	steady state temperature scale for medium and low Rayleigh number regime (K)
T_{bw}	temperature scale at t_w (K)
T_{bw^+}	temperature scale after t_w (K)
u, v	horizontal and vertical velocity components (m/s)
U, V	dimensionless horizontal and vertical velocity components
v_s	steady state velocity scale for distinct thermal boundary layer (m/s)
v_w	velocity scale at t_w (m/s)
v_1	velocity scale at $Pr^{-1}t_w$ (m/s)
v_{f1}, v_{f2}	steady state velocity scale for medium and low Rayleigh number regime (m/s)
V_1	dimensionless velocity scale at $Pr^{-1}t_w$
W_g	air gap width (m)
x, y	horizontal and vertical coordinates (m)
X, Y	dimensionless horizontal and vertical coordinates

Greek symbols

α	thermal diffusivity (m^2/s)
β	coefficient of thermal expansion (1/K)
δ_T	thermal boundary layer thickness (m)
δ_{Ts}	thermal boundary layer thickness at steady state (m)
ΔT	temperature difference between the wall and the free stream (K)
ΔP_{Ha}, ΔP_s	pressure loss along the air path and the stack pressure (Pa)
θ	dimensionless temperature
θ_{bw^+}	dimensionless temperature scale after t_w
υ	kinematic viscosity (m^2/s)
ρ	density (kg/m^3)
τ	dimensionless time

found in passive heating, cooling and ventilation systems in buildings and in electronic cooling devices. A passive system such as solar chimney attached to a building serves as an excellent ventilation means that relies only on a natural driving force, i.e. the energy from the sun. The adoption of solar chimney for building ventilation is increasing due to its effectiveness and environment-friendly features, and solar chimney has attracted increasing research attention in recent years (see for example Nouanégué and Bilgen, 2009; Punyasompun et al., 2009; Zamora and Kaiser, 2009; Siva Reddy et al., 2011). Solar chimney may be described as an asymmetrically heated air channel with air flow constrained between two vertical walls (glazing and absorber), where the air movement is due to the buoyancy force generated by solar heating.

For natural ventilation applications, the mass flow rate resulting from the temperature difference between the absorber wall and the ambient is of prime importance, and it provides an indication of the effectiveness of the solar chimney system. In order to predict the performance of the solar chimney system, various lumped parameter models and simplified analytical models have been developed (see for example Bansal et al., 1993; Ong, 2003; Bassiouny and Koura, 2008). These models are mostly based on simple mass and energy conservations with bulk flow assumptions and do not account for the detailed flow development within the solar chimney channel. For relatively high Rayleigh number applications, it is expected that a distinct thermal boundary layer (i.e. the thickness of the thermal boundary layer is smaller than the air gap width – the distance between the glazing and absorber) develops in the solar chimney along the heated wall (absorber) as a consequence of the solar heating. In this context, the bulk flow assumption, i.e. a uniform variation of the flow velocity and temperature inside a solar chimney channel is not appropriate. Therefore, concerns have been raised about the suitability of these models for predicting the air flow rate in solar chimney (Chen et al., 2003). On the other hand, by considering the thermal boundary layer flow through a transient scale analysis, a comprehensive understanding about the flow development in a solar chimney channel can be developed, leading to a better prediction of the mass flow rate through the solar chimney.

In a recent numerical study (Khanal and Lei, 2012), the present authors presented some simple scaling for evaluating the maximum steady-state mass flow rate through the solar chimney under the high-Rayleigh number scenario, but the detailed flow development was not reported. With the exception to the above-mentioned work of the present authors, to the best of our knowledge no scaling analysis entirely dedicated to solar chimney ventilation has been reported in the open literature, although scale analysis of vertical thermal boundary layer flows has been reported extensively (see for example Patterson and Imberger, 1980; Lin and Armfield, 2005b; Armfield et al., 2007; Patterson et al., 2009). Moreover, all these reported studies are relevant to freely developing thermal boundary layers. In the solar chimney application, the development of the thermal boundary layer flow adjacent to the absorber wall may be constrained by the presence of the glazing, and the thermal boundary layer may expand to the full air gap width. This situation prevails in solar chimney when it operates under low solar energy input or when the air channel is narrow. Because of this fundamental departure from the freely developing boundary layers, separate scaling relations are needed to characterize the thermal boundary layer development in solar chimney. These are the motivations behind the present study. The present