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Modelling turbulent spots in swept boundary layers

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ABSTRACT

A computational technique is presented for determining the fully 3-d viscid unsteady perturbation to a non-developing laminar swept boundary layer. For zero pressure gradient, unswept boundary layers, the perturbation method reveals a strongly three dimensional flow within the turbulent spot and its associated calmed region which is very similar to that observed in experiments and full DNS calculations. The perturbation method cannot predict turbulent motion but nevertheless provides a simple yet accurate means of studying and understanding the development of turbulent spot geometry. The most influential flow feature is the horseshoe vortex observed in the fluctuation velocity field, which is responsible for delivering the fluid found in the calmed region between its trailing legs. The upwards flow around the outer periphery of the vortex is also responsible for delivering low momentum fluid to the spot, but additional high momentum fluid also enters the spot from its rear through the downward sweeping motion of fluid between the vortex legs. The effect of an adverse streamwise pressure gradient is to increase the size of the spot and calmed region whereas a favourable pressure gradient has the opposite effect. When sweep is introduced to the boundary layer the spot is skewed for all non-zero pressure gradients, but the changes in size of the spot and calmed region due to pressure gradient are reduced. For favourable pressure gradients the skew increases monotonically with sweep, but this is not the case for adverse pressure gradients where the effect of sweep is more complex.

1. Introduction

Turbulence and transition modelling remains a critical factor in the accurate prediction of many engineering flows. Direct numerical simulation provides accurate solutions to transitional and turbulent flow problems, but it is prohibitively expensive for all but the simplest of engineering flows and is likely to remain so for many decades. The prediction of transitional and turbulent effects is therefore commonly achieved using models which are derived either from experimental or DNS results, however these results are only available for a relatively small number of flow conditions. For boundary layer flows nearly all models are derived from data for 2-d boundary layers on flat plates, as data for more complex 3-d boundary layers on curved surfaces are sparse. Obtaining sufficient data to formulate models for a sufficient variety of 3-d boundary layers is however a daunting task through experiment or DNS. Unfortunately most engineering boundary layers are 3-d in nature and rarely flow over flat surfaces and so there is a requirement to extend current predictive methods to more complex 3-d boundary layers.

The majority of current transition models rely on correlations for the start and end of transition based on carefully executed wind tunnel experiments on flat plates. The start of transition correlation derived by Abu-Ghannam and Shaw (1980) is commonly used, but their very simple end of transition correlation is frequently found to be insufficiently accurate. Gostelow recognised that the transition length could be predicted accurately if the rate at which turbulent spots are generated and the subsequent spreading angle of the spots is correlated from experimental data. To this end he combined results from previously published work on turbulent spot spreading angles with his own new experimental data Gostelow et al. (1995) to obtain a correlation for spot spreading angles over a full range of pressure gradients for attached boundary layers. Solomon et al. (1995) successfully used this new correlation to successfully predict the ERCOFTAC test cases Savill (1991). However, this success may be partly attributable to the fact that the ERCOFTAC test cases are for very similar two dimensional flat plate boundary layers to those used to derive the correlation. The significant differences in transition which occur for 3-d boundary layers are well documented. For example, the effect on transition of concave curvature, due to the presence of Taylor Gortler vortices, has been observed by several researchers e.g. Zhang et al. (1995) and Volino and Simon (1997). Johnson (2007) formulated a transition model for these boundary layers, but this was limited to zero streamwise pressure gradients and transition models for any 3-d boundary layer remain rare. Similarly there have been very few studies of turbulent spots in 3-d boundary layers. Jahanmiri et al. (1996) have shown through experiment that the

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Nomenclature

p'	fluctuating pressure
Re_δ	$\left(=\frac{U\delta}{\nu}\right)$ boundary layer thickness Reynolds number
Re_θ	$\left(=\frac{U\theta}{\nu}\right)$ momentum thickness boundary layer Reynolds number
t	Time
T	$\left(=\frac{Ut}{\delta}\right)$ dimensionless time
u	steady velocity in x direction
U	freestream time mean velocity
u', v', w'	fluctuating velocities in x, y and z directions
v'_0	amplitude of initiating pulse at x, y, z origin and $t = 0$
w	steady velocity in z direction
x	co-ordinate measured from plate leading edge
y	co-ordinate measured from wall
z	spanwise co-ordinate
X, Y, Z	$\left(=\frac{x}{\delta}, \frac{y}{\delta} \text{ and } \frac{z}{\delta}\right)$ dimensionless coordinates
X_f, Z_f	dimensionless co-ordinates in freestream and cross flow directions
α	sweep flow angle
δ	boundary layer thickness
θ	boundary layer momentum thickness
λ	$\left(=\frac{\delta^2}{\nu}\frac{dU}{dx}\right)$ Pohlhausen pressure gradient parameter
λ_θ	$\left(=\frac{\theta^2}{\nu}\frac{dU}{dx}\right)$ Thwaites pressure gradient parameter
ν	kinematic viscosity
ρ	Fluid density

Subscripts

c	cos coefficient
max	spatial maximum value
P	pulse width
s	sin coefficent
t	t derivative
T	T derivative
x, y, z	x, y and z derivatives
X, Y, Z	X, Y and Z derivatives

development of a turbulent spot changes markedly when spanwise convergence or divergence is introduced to the boundary layer. There have however been no studies, to the author's knowledge, of the development of turbulent spots on concave surfaces. Evidence from previous work therefore demonstrates that the transition process is significantly different in 2-d and 3-d boundary layers and therefore the ability of 2-d transition models to predict transition for 3-d boundary layers must be questioned.

Many researchers (e.g. Glezer et al. (1989), Gutmark and Blackwelder (1987), Itsweire and Van Atta (1984), Katz et al. (1990) and Sankaran et al. (1988)) have used a variety of experimental techniques to study induced turbulent spots in 2-d laminar boundary layers. Ensemble averaging of a large number of spot realisations has revealed that the basic structure is that of a single horseshoe vortex tube (Itsweire and Van Atta (1984)). The spot itself has a planform similar to a triangular arrowhead, although a streamwise pressure gradient can lead to changes in this shape (Katz et al. (1990) and Gostelow et al. (1995)). The trailing edge of the spot is almost invariant with both distance from the wall and spanwise position and has a convection velocity close to half the freestream velocity. The leading edge develops an overhang (Gutmark and Blackwelder (1987)) which travels at approximately 90% of the freestream velocity distant from the wall, but at a lower velocity close to the wall. Substructures, described as eddies, horseshoe or lambda vortices, have been identified within the spot (Sankaran et al. (1988)) as being responsible for the generation of turbulence. The spot appears to grow through the addition of further substructures rather than the growth in the substructures themselves. These observations have helped to describe the structure of the turbulent spot, but as yet a complete understanding of the flow structure within the turbulent spot has not resulted.

Johnson (1998, 1999) has shown previously that relatively simple linear perturbation techniques can provide accurate predictions of the characteristics governing the development of turbulent spots in two dimensional boundary layers. The objective of the current work is to use these numerical techniques to predict turbulent spot development in swept boundary layers and hence determine whether the spot development and hence the transition length can be accurately predicted using current correlations derived from 2-d boundary layers.

2. Computational method

2.1. Equations of motion

In the current work, the unsteady flow is assumed to be a small linear perturbation to the time mean flow. For this reason only the primary instabilities within the flow are determined rather than full breakdown to turbulence. Nevertheless the characteristics of the linearly disturbed region are very similar to those of the turbulent spot as shown previously for Poiseuille flow by Li and Widnall (1989) and 2-d boundary layers by Johnson (1998,1999, 2001).

The two co-ordinate systems used here for the swept plate are shown in Fig. 1. The x–z co-ordinates are aligned with the plate, whereas the x_f–z_f system is aligned with the freestream velocity U. The plate sweep angle is α. The time mean flow is considered to be inviscid and hence the boundary layer profile is invariant across the plate. The steady flow velocity is therefore parallel with the plate and constant in magnitude and direction on each x–z plane but both its magnitude and direction vary in the y direction normal to the plate. The velocity profile is therefore considered to be that for a non-developing boundary layer on an infinitely wide plate where a pressure gradient exists only in the x direction. The u velocity component profile (in the x direction) can then be approximated by the Pohlhausen polynomial

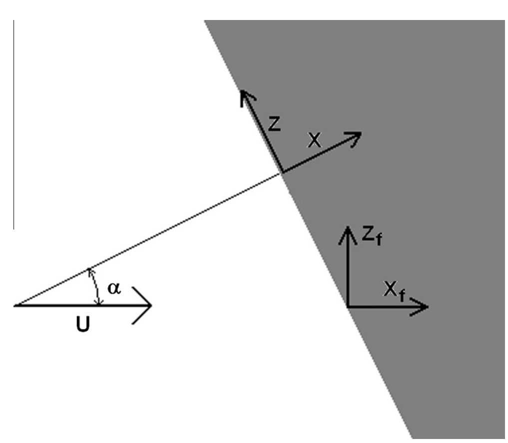

Fig. 1. Co-ordinate systems.