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Simulation of forced deformable bodies interacting with two-dimensional incompressible flows: Application to fish-like swimming

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ABSTRACT

We present an efficient algorithm for simulation of deformable bodies interacting with two-dimensional incompressible fluid flows. The temporal and spatial discretizations of the Navier–Stokes equations in vorticity stream-function formulation are based on classical fourth-order Runge–Kutta scheme and compact finite differences, respectively. Using a uniform Cartesian grid we benefit from the advantage of a new fourth-order direct solver for the Poisson equation to ensure the incompressibility constraint down to machine zero over an optimal grid. For introducing a deformable body in fluid flow, the volume penalization method is used. A Lagrangian structured grid with prescribed motion covers the deformable body which is interacting with the surrounding fluid due to the hydrodynamic forces and the torque calculated on the Eulerian reference grid. An efficient law for controlling the curvature of an anguilliform fish, swimming toward a prescribed goal, is proposed which is based on the geometrically exact theory of nonlinear beams and quaternions. Validation of the developed method shows the efficiency and expected accuracy of the algorithm for fish-like swimming and also for a variety of fluid/solid interaction problems.

1. Introduction

The quantification and simulation of the flow around biological swimmers is one of the challenges in fluid mechanics (Sotiropoulos and Yang, 2014). At the same time bio-inspired design of swimming robots are in growth (El Rafei et al., 2008). The costs of experimental studies (Belkhiri, 2013) lead the researchers to develop efficient predictive numerical algorithms for hydrodynamic analyses of fish swimming. Difficulties of numerical simulations of fish-like swimming are due to different reasons. One problem is efficient quantification of the kinematics of different species (more than 32,000) which seems to be far from the simple laws proposed in different studies. Efficient simulation of incompressible flows is also an important problem, where the efficiency of the elliptic solver is crucial. The third bottleneck in numerical simulations of fish-like swimming is the coupling of the fluid solver with deformable, moving and rotating bodies. Fishes swim by exerting force and torque against the surrounding water. This is normally done by the fish contracting muscles on either side of its body in order to generate moving waves from head to tail. These waves generally are getting larger as they go toward the tail (Wikipedia contributers, 2014). The resultant force exerted on the water by such motion generates a force (even oscillatory) which pushes the fish forward. Most fishes generate thrust moving their body and fins. In general these movements can be divided into *undulatory* and *oscillatory* motions. Mechanisms of locomotion using body and fins are divided into groups that differ in the fraction of their body that is displaced laterally (Breder, 1926). *Anguilliform* swimmers are long and slender, in which there is little increase in the amplitude of the flexion wave as it passes along the body. In *carangiform* swimmers, there is a more remarkable increase in wave amplitude along the body with the vast majority of the work being done by the rear half of the fish. In *thunniform* fishes almost all the lateral movement is in the tail. *Ostraciiform* fishes have no appreciable body wave when they employ caudal locomotion, only the tail fin itself oscillates rapidly to create thrust. However there are other minorities (Wikipedia contributers, 2014). The tail beat creates a reversed Kármán street of vortices and generates thrust, leaving thus a momentumless wake back. By varying the frequency and amplitude of the oscillation a variety of wakes, like classical Kármán, two pairs (2P) (Van Rees et al., 2013), two pairs plus two single (2P+2S), etc. Schnipper et al. (2009) can be observed (Williamson and Roshko, 1988). Anguilliform fishes add a constant curvature to their backbone for turning, i.e., they use their body like a rudder for torque generation. Yeo

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et al. (2010) studied numerically the straight swimming/cruising and sharp turning manoeuvres in two-dimensions. It was shown by Yeo et al. (2010) that a carangiform-like swimmer execute a sharp turn through an angle of 70° from straight coasting within a space of about one body length. Gazzola et al. (2012) investigated the C-start escape patterns of a larval fish by using a remeshed vortex particle method and the volume penalization. The deformation of the fish, based on the mid-line curvature values, is optimized via an evolutionary strategy by Hansen et al. (2003) to maximize the escape distance. Bergmann and Iollo (2011) performed numerical simulations of fish rotation and swimming toward a prescribed goal. They considered the average profile of the fish backbone aligns over a circle with an estimated radius to perform a rotation. The radius of the circle tends to infinity ($r \to \infty$) in a forward gait. The considered fish by Bergmann and Iollo (2011) is constructed by a complex valued mapping like the Kutta–Joukowski transform superposed to the fish backbone with prescribed undulatory motion. Here we will present a simple law for turning control of an anguilliform fish. Our rotation control law (Bontoux et al., 2014) is similar to that presented by Yeo et al. (2010), and Bergmann and Iollo (2011), in which the feedback is based on the angle between the line-of-sight and the direction of surge. But instead of adding a radius to the backbone, we envisage to use curvature which seems to be more efficient. We use the method proposed by Boyer et al. (2006) which is based on quaternions for efficient description of the fish backbone kinematics.

We apply the rotation control to two-dimensional swimming. Even if due to the shape and deformation style of the fish-like swimmers the surrounding flow is fully three dimensional, most of the fundamental features of swimming are included in two-dimensional analyses. For incompressible flows the Navier–Stokes equations can be reformulated in terms of vorticity-velocity (Gazzola et al., 2011) or vorticity stream-function (Spotz and Carey, 1995). For two-dimensional problems the vorticity formulation is reduced to a scalar valued evolution equation. Hence only the vorticity transport equation has to be advanced in time. The choice of finite differences in this paper is related to the use of an immersed boundary method in which a Cartesian grid can be used. Therefore the use of finite differences is efficient and straightforward. Among finite difference methods high-order compact discretizations, (Hirsh, 1975; Lele, 1992), are more advantageous in terms of accuracy and reasonable cost. We refer to Abide and Viazzo (2005) and Boersma (2011) for high-order compact discretizations of the incompressible Navier–Stokes equations in primitive variables and to Bontoux et al. (1978), Roux et al. (1980), and Spotz and Carey (1995) for compact high-order solutions of the vorticity and stream-function formulation. Solving the incompressible Navier–Stokes equations typically implies an elliptic Poisson equation which is the most time consuming part of the algorithm. Direct methods like diagonalization or FFT based solvers can be used. Iterative methods, namely, point successive over relaxation (PSOR) with read-black sweeper, multigrid or Krylov subspace solvers are other alternatives. Using high-order discretizations iterative methods are less attractive because the resulted matrices are less sparse, thus the rates of convergence are slow. However iterative methods can cover all types of boundary conditions, we refer to Spotz and Carey (1995) for a fourth-order compact discretization of the Poisson equation. On the other hand, in direct methods the memory limitation is restrictive for simulations on a fine grid. Therefore decoupling of the directions by FFT based methods can be advantageous, even if this method implies some limitations in the boundary conditions. We propose a direct fourth-order solver for the Poisson equation which is a combination of a compact finite difference with a sine FFT. The main advantages of our method are fourth-order accuracy, efficiency, the possibility to parallelize and convergence down to zero machine precision over an optimal grid. Other advantages and limitations of the proposed solver are discussed in the paper. A difficulty in numerical simulations of fish swimming is the analysis of fluid/solid interaction, which can be handled by strong or loose coupling according to implicit or explicit time advancement, cf. (Sotiropoulos and Yang, 2014) for a detailed discussion. We use the volume penalization method, known also as Darcy-Brinkmann penalization (Brinkmann, 1947), proposed by Arquis and Caltagirone (1984), Angot et al. (1999) and Khadra et al. (2000), which belongs to the diffuse-interface immersed boundary methods (IBMs). It consists of modeling the immersed body as a porous medium, thus getting rid of the Dirichlet boundary conditions by considering both the fluid and the body as one domain with different permeabilities. So one can consider a rectangular solution domain in which the body is immersed and can even move. The penalization method leads to between first and second order accuracy near the body and is an efficient method in dealing with deformable, moving and rotating bodies immersed in a fluid. A development to deal with rigid bodies colliding with each other in incompressible flows is performed by Coquerelle and Cottet (2008). An extension to include elasticity of the solid interacting with fluid via the volume penalization method is represented by Engels et al. (2013). One advantage of this class of penalization schemes for fluid–structure interaction problems is that it enables the use of time and space adaptivity via multiresolution analysis as recently demonstrated by Gazzola et al. (2014) and Ghaffari et al. (2014). We refer to the review of Mittal and Iaccarino (2005) for a complete classification and description of immersed boundary methods.

In the present work, we will focus on some numerical aspects of efficient turning laws for anguilliform swimmers, a topic which is less studied so far. To this end the geometrically exact theory of nonlinear one-dimensional beams based on quaternions (Boyer et al., 2006) is adapted to the backbone kinematics description. Starting by the code developed by Sabetghadam et al. (2009) we apply compact finite differences to the vorticity stream-function formulation of the Navier–Stokes equations including the penalization term. An efficient direct method is presented for solving the Poisson equation. Thus different numerical aspects of the algorithm like accuracy in space and the error introduced by the penalization method will be examined. The code is developed in FORTRAN and is open access (Ghaffari). The paper is organized as follows. First our methodology including the governing equations, discretization, kinematics of an anguilliform swimmer and the algorithm for fluid interaction with forced deformable bodies will be presented. Next a validation of the algorithm will be carried out, the errors will be assessed and their convergence will be studied. Then the results for swimming and rotation control are reported. Finally, the results will be discussed and some guidelines for future works will be addressed.

2. Methodology

2.1. Governing equations of incompressible flow

The governing equations of incompressible flows are the Navier–Stokes equations. In two-dimensional problems the vorticity and stream-function formulation in comparison to the primitive variable formulation has the advantage that it not only eliminates the pressure, but also ensures a divergence-free velocity field (mass conservation, i.e., $\nabla \cdot \mathbf{u} = 0$) if the Poisson Eq. (2) is properly satisfied (Bontoux et al., 1978; Roux et al., 1980). One encounters two scalar valued quantities, i.e., the vorticity ω and the stream-function ψ, instead of the velocity vector and the pressure field, thus it makes the computations more efficient. With this formulation, it is possible to use a collocated grid without adding any explicit numerical dissipation, which reduces the arithmetics