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Recent progress in the development of the Elliptic Blending Reynolds-stress model

Rémi Manceau

Department of Applied Mathematics, CNRS—University of Pau—Inria, avenue de l'université, 64013 Pau, France

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ABSTRACT

The Elliptic Blending Reynolds Stress Model (EB-RSM), originally proposed by Manceau and Hanjalić (2002) to extend standard, weakly inhomogeneous Reynolds stress models to the near-wall region, has been subject to various modifications by several authors during the last decade, mainly for numerical robustness reasons. The present work revisits all these modifications from the theoretical standpoint and investigates in detail their influence on the reproduction of the physical mechanisms at the origin of the influence of the wall on turbulence. The analysis exploits recent DNS databases for high-Reynolds number channel flows, spanwise rotating channel flows with strong rotation rates, up to complete laminarization, and the separated flow after a sudden expansion without and with system rotation. Theoretical arguments and comparison with DNS results lead to the selection of a recommended formulation for the EB-RSM model. This formulation shows satisfactory predictions for the configurations described above, in particular as regards the modification of the mean flow and turbulent anisotropy on the anticyclonic or pressure side.

1. Introduction

Since the pioneering work of Chou (1945), second-moment closures of the RANS (Reynolds-Averaged Navier–Stokes) equations have been essentially founded upon a theoretical framework valid in homogeneous or weakly inhomogeneous turbulence. However, in the vicinity of solid boundaries, turbulence exhibits specific properties that are drastically different from those observed in homogeneous flows. In turbulence modelling, some of these features cannot be overlooked: scales separation is not sufficient to neglect the influence of molecular viscosity on the large, energetic, scales and of large-scale anisotropy on the small, dissipative, scales; the blockage of the wall-normal fluctuations is at the origin of a strong departure from isotropy, and leads to a two-component state of turbulence at the wall; through pressure fluctuations, turbulence loses its local character and feels the presence of the wall in a region much larger than the viscosity affected region.

These features are not accounted for by standard models based on weakly inhomogeneous assumptions, which are consequently not valid in the near-wall region and require the use of wall functions. Various approaches have been proposed in the literature to extend the validity of these models to near-wall regions (for a review of the state of the art, see Hanjalić and Launder, 2011). One of the most successful methods, proposed by Durbin (1991), consists in solving an *Elliptic Relaxation* equation for the velocity–pressure gradient correlation tensor involved in the Reynolds-stress transport equation, in order to account for the crucial wall blockage. This method is based on solid theoretical foundations and is supported by DNS data (Manceau et al., 2001), but introduces 13 additional differential equations (for the Reynolds stresses, the dissipation rate and the six components of the velocity–pressure gradient correlation) and is numerically stiff because of the boundary conditions of the Elliptic Relaxation equations. In order to reduce the number of equations and to circumvent numerical difficulties, Manceau and Hanjalić (2002) proposed the *Elliptic Blending* approach, in which the six Elliptic Relaxation equations are replaced by a single equation for a wall-sensitive nondimensional scalar α, with the simple boundary conditions $\alpha = 0$ at the wall.

However, the first applications of the Elliptic Blending strategy to more complex configurations (Shin et al., 2003; Thielen et al., 2005; Manceau, 2005) highlighted some deficiencies of the model, which is at the origin of various modifications of the original model that are summarized in Section 4. These modifications mainly affect the formulations of the model for the velocity–pressure gradient tensor, the blending function used to migrate from the near-wall form to the weakly inhomogeneous form of the model and the

E-mail address: remi.manceau@univ-pau.fr

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dissipation equation. On the one hand, such a variability is evidence that the model is alive and applied to practical configurations, but, on the other hand, it is a source of confusion, since the appellation *Elliptic Blending Reynolds Stress Model* (EB-RSM) actually refers to numerous, slightly different models. My personal experience has shown that new users are often confused by this variability and request information about the version they must start with.

Consequently, the main objective of the present article, after more than a decade of developments and applications of the approach by several authors, is to revisit all these modifications, from their theoretical justification to their practical application, in order to investigate their relevance in terms of representation of the near-wall physics. The present work is thus targeted at formulating a *reference* EB-RSM, with a proper set of coefficients. Since validating this model against the wide range of applications already treated with EB-RSM in the past would be a formidable task, the present paper is focused on the detailed investigation of the reproduction of the main physical mechanisms in a few canonical channel and separated flows, including cases with spanwise rotation, which are relevant to many applications, from turbomachinery to geophysical flows, using, among others, very recent DNS databases that have not been exploited yet in the context of RANS modelling (Lozano-Durán and Jiménez, 2014; Lamballais, 2014).

2. Physical background

As a preliminary to the introduction of turbulence models dedicated to the reproduction of the near-wall region, it is useful to summarize the various effects exerted by solid boundaries on the turbulent field and how they can be accounted for in Reynolds stress models. In particular, the distinction between viscous and non-viscous effects is favourable to the interpretation of the role played by the Elliptic Relaxation and Elliptic Blending approaches.

(i) Due to the no-slip boundary condition linked to the viscous character of the fluid, and the associated strong velocity gradients, a peak of turbulence production occurs in the near-wall region. In the framework of near-wall models, this effect is easily reproduced by accounting for the exact boundary condition for the mean velocity $U_i = 0$. However, it should be pointed out that the quasi-homogeneous hypothesis (Chou, 1945), i.e., the local approximation of the mean velocity gradient by a constant in the rapid part of the redistribution term, is consequently not valid in the near-wall region (Bradshaw et al., 1987). It will be seen below that this approximation is not used in the near-wall region in the Elliptic Relaxation and Elliptic Blending models.

(ii) The no-slip condition applied to the velocity fluctuations leads to a viscous damping of the Reynolds stresses, which makes necessary the accounting of viscous terms in the Reynolds-stress transport equations: the Reynolds stresses behave as y^2, if y represents the direction normal to a wall located in $y = 0$ (as shown below, the behaviour in y^n, with $n > 2$, of some components is due to the blockage effect). The boundary condition $\overline{u_i u_j} = 0$ is not sufficient to impose this behaviour, which requires the correct reproduction of the balance between viscous diffusion and dissipation in the Reynolds stress transport equations in the vicinity of the wall. This requirement is consequently linked to the correct modelling of the dissipation tensor: since the viscous damping also suppresses the scale separation between energetic and dissipative structures, the anisotropy of the dissipation tensor cannot be neglected. In practice, the correct asymptotic behaviour is obtained in near-wall Reynolds-stress models by choosing a model for the dissipation tensor that satisfies

$$\lim_{y\to 0} \varepsilon_{ij} = 2\nu \lim_{y\to 0} \frac{\overline{u_i u_j}}{y^2}. \tag{1}$$

(iii) In the case of an incompressible flow, the continuity equation implies that the fluctuating velocity normal to an impermeable surface, v, behaves as y^{n+1}, where n is the exponent describing the asymptotic behaviour (y^n) of the tangential components u and w (for details, see Appendix A). As a consequence, the Reynolds stress component $\overline{v^2}$ behaves as y^{2n+2}, and is asymptotically negligible compared to the other two normal stresses, $\overline{u^2}$ and $\overline{w^2}$, which behave as y^{2n}. This effect is purely kinematic, independent of the viscous character of the fluid, and is observed for solid walls as well as free-slip surfaces (Yokojima and Shima, 2010). Therefore, for models that are able to reproduce the correct behaviour of turbulence in the presence of solid walls, the ambiguous designation *low-Reynolds number* models should be avoided and replaced by *near-wall* models. For a free-slip surface, components $\overline{u^2}$ and $\overline{w^2}$ behave as y^0 and $\overline{v^2}$ as y^2; for a no-slip surface, this asymptotic behaviour is changed to y^2 and y^4, respectively. In all cases, the most important effect to take into account is the fact that $\overline{v^2}$ is negligible compared to the other normal stresses, such that turbulence reaches a *two-component limit* in the vicinity of the surface. In the case of a no-slip wall, one of the main difficulties lies in the particular scales of this phenomenon, linked to the non-locality of the fluctuating pressure, at the origin of a sensitivity of turbulence to the blockage effect up to a distance to the wall much larger than the thickness of the viscous sublayer. The layer in which $\overline{v^2}$ is affected by this effect is called the *source layer* (Hunt and Graham, 1978; Calmet and Magnaudet, 2003), *surface-influenced layer* (Brumley and Jirka, 1987) or *blockage layer* (Campagne et al., 2009). In order to model this blockage effect, the correct reproduction of the near-wall balance between viscous diffusion and dissipation, Eq. (1), is not sufficient: the asymptotic behaviour of the velocity–pressure gradient correlation term must be accounted for. This is the main purpose of the Elliptic Relaxation (Durbin, 1991) and Elliptic Blending (Manceau and Hanjalić, 2002) methods, briefly described below.

(iv) The presence of a wall induces an increase of pressure fluctuations, called the *wall echo* effect, which can be explained by considering the Poisson equation for the fluctuating pressure p

$$\nabla^2 \mathrm{p} = -2\rho \frac{\partial U_i}{\partial x_j}\frac{\partial u_j}{\partial x_i} - \rho \frac{\partial u_i}{\partial x_j}\frac{\partial u_j}{\partial x_i} + \rho \overline{\frac{\partial u_i}{\partial x_j}\frac{\partial u_j}{\partial x_i}} \equiv S_p \tag{2}$$

in a semi-infinite domain bounded by a wall located at $y = 0$. In order to separate viscous and non-viscous effects, the fluctuating pressure is split into $\mathrm{p} = p + p^s$, where p^s is Stokes' pressure, defined as the solution of the problem

$$\nabla^2 p^s = 0, \tag{3}$$

$$\frac{\partial p^s}{\partial y} = \rho\nu \frac{\partial^2 v}{\partial y^2}. \tag{4}$$

In contrast to the blockage and wall echo effects, the appearance of Stokes' pressure is due to viscosity, and as shown by Kim (1989), it is negligible. The remaining fluctuating