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Turbulent thermal boundary layers with temperature-dependent viscosity

Jin Lee [a], Seo Yoon Jung [b,1], Hyung Jin Sung [a,*], Tamer A. Zaki [b,2]

[a] Department of Mechanical Engineering, KAIST, 291 Daehak-ro, Yuseong-gu, Daejeon 305-701, Republic of Korea
[b] Department of Mechanical Engineering, Imperial College London, South Kensington Campus, London SW7 2AZ, UK

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ABSTRACT

Direct numerical simulations (DNS) of turbulent boundary layers (TBLs) over isothermally heated walls were performed, and the influence of the wall-heating on the thermal boundary layers was investigated. The DNS adopt an empirical relation for the temperature-dependent viscosity of water. The Prandtl number therefore changes with temperature, while the Péclet number is constant. Two wall temperatures ($T_w = 70$ °C and 99 °C) were considered relative to $T_\infty = 30$ °C, and a reference simulation of TBL with constant viscosity was also performed for comparison. In the variable viscosity flow, the mean and variance of the scalar, when normalized by the friction temperature deficit, decrease relative to the constant viscosity flow. A relation for the mean scalar which takes into account the variable viscosity is proposed. Appropriate scalings for the scalar fluctuations and the scalar flux are also introduced, and are shown to be applicable for both variable and constant viscosity flows. Due to the modification of the near-wall turbulence, the Stanton number and the Reynolds analogy factor are augmented by 10% and 44%, respectively, in the variable viscosity flow. An identity for the Stanton number is derived and shows that the mean wall-normal velocity and wall-normal scalar flux cause the increase in the heat transfer coefficient. Finally, the augmented near-wall velocity fluctuations lead to an increase of the wall-normal scalar flux, which contributes favorably to the enhanced heat transfer at the wall.

1. Introduction

Turbulent flows of liquids over heated walls are of practical importance in many engineering problems such as heat exchangers and nuclear reactors. For common liquids including water, viscosity decreases with increasing temperature. When a large temperature difference between the wall and the free stream is established, the resulting temperature gradient near the wall causes a gradual change in viscosity. Although turbulence modification due to the temperature-dependent viscosity in heated flows was previously addressed (e.g. Zonta et al., 2012; Lee et al., 2013), the effect of the viscosity variation on the thermal boundary layer and scalar transport has not received a similar level of attention.

A number of studies were devoted to numerical simulations of turbulent thermal boundary layer flows, but have generally assumed constant fluid properties and in particular the Prandtl number, *Pr*. For example, based on direct numerical simulations (DNS), Kong et al. (2000) demonstrated the similarity between the wall-normal heat flux and the Reynolds stresses, which underlies the correlation between the temperature and the streamwise velocity fluctuations. Most of the earlier studies concentrated on the effect of different, but constant, *Pr* on the mean scalar quantities and scalar fluxes. Tiselj et al. (2001) and Kozuka et al. (2009) investigated scalar transfer in turbulent channel flows at different Prandtl numbers. Abe et al. (2004) examined the Reynolds-number (*Re*) effect on the scalar transfer as well as the *Pr*-effect ($Pr = 0.71$ and 0.025) in channel flows. They showed that the scalar-flux fluctuations are increased in high-*Re* flows due to augmented turbulence activity. Transitional and turbulent thermal boundary layers were studied by Li et al. (2009) and Wu and Moin (2010). The former work investigated the effects of thermal boundary conditions and the Prandtl number. Wu and Moin (2010) provided the statistics of a spatially developing flow up to relatively higher-*Re*. These studies focused on the scalar transport and contributed to our understanding of turbulence structures including the velocity and temperature fluctuations in flows with various thermal boundary conditions and Prandtl numbers.

* Corresponding author. Tel.: +82 42 350 3027; fax: +82 42 350 5027.
E-mail address: hjsung@kaist.ac.kr (H.J. Sung).

[1] Present address: Advanced Reactor Development Institute, KAERI, 989-111, Daedeok-daero, Yuseong-gu, Daejeon 305-353, Republic of Korea.

[2] Present address: Department of Mechanical Engineering, Johns Hopkins University, Baltimore, MD 21218, USA.

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The above numerical studies assumed constant fluid viscosity or, equivalently, Prandtl number. Few researchers took into account the temperature-dependence of viscosity, e.g. Wall and Wilson (1997) and Sameen and Govindarajan (2007). Most previous research on the influence of viscosity stratification focused on the stability of laminar boundary layers and not on TBLs. We herein address this gap and consider the case of temperature-dependent viscosity, where the Prandtl number varies spatially within the TBL. One relevant study was the recent work by Zonta et al. (2012), who performed DNS of turbulent channel flow with wall heating. They examined the effect of inhomogeneous viscosity and found that turbulence production and dissipation of the wall-bounded flow were significantly altered. Their work did not, however, consider the heating of spatially developing flows.

Recently, the mechanism of skin-friction reduction owing to the temperature-dependent viscosity was studied by Lee et al. (2013). That work demonstrated weakening of the outer vortices and enhanced fine-scale motions near the heated walls. The present work is a continuation of the study by Lee et al. (2013). We examine the transport of scalars, such as temperature or concentration, owing to the viscosity gradient. DNS data of forced convection in TBLs with temperature-dependent viscosity are utilized. The free-stream fluid is assumed to be water at 30 °C, which corresponds to $Pr = 5.4$. Using an empirical model of the water viscosity, two wall temperatures (70 °C and 99 °C) are considered in order to establish the viscosity difference; namely, moderately heated (MH) and strongly heated (SH) walls. For comparison, a conventional passive scalar simulation, herein referred to as constant viscosity (UH; the term was 'unheated' in Lee et al., 2013), is also considered.

2. Numerical details

The temperature-dependent viscosity of water is defined by the Arrhenius-type viscosity model (White, 2006). In order to isolate the effect of the viscosity variation alone, the thermal diffusivity (α) and density (ρ) are assumed to be constant and are set by the free-stream temperature. The present simulation belongs to the forced convection regime, $Gr/Re^2 \ll 1$, where Gr is the Grashof number and Re is the Reynolds number.

The governing equations for an incompressible flow with temperature-dependent viscosity are

$$\frac{\partial u_i}{\partial x_i} = 0, \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re_{\theta_{in}}} \frac{\partial}{\partial x_j} \left[\nu_R \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right], \tag{2}$$

and

$$\frac{\partial \Theta}{\partial t} + u_j \frac{\partial \Theta}{\partial x_j} = \frac{1}{Re_{\theta_{in}} Pr_\infty} \frac{\partial^2 \Theta}{\partial x_j^2}. \tag{3}$$

The velocity components in the streamwise (x), wall-normal (y) and spanwise (z) directions are u, v and w, respectively, and p is the kinematic pressure. The non-dimensional temperature deficit, herein referred to as the scalar, is defined as $\Theta = (T - T_w)/(T_\infty - T_w)$. Subscripts w and ∞ denote variables at the wall and in the free stream, respectively. The ratio of the local to the free-stream viscosity is $\nu_R \equiv \nu(T)/\nu_\infty$. Note that the physical temperature (T) is used to determine the viscosity ratio. The Reynolds and Prandtl numbers in the governing equations are $Re_{\theta_{in}}$ ($\equiv U_\infty \theta_{in}/\nu_\infty$) = 1240 and Pr_∞ ($\equiv \nu_\infty/\alpha$) = 5.4, respectively. The numerical method for the solution of the Navier-Stokes equations is summarized in Zaki et al. (2010), and was previously applied in the DNSs of various transitional (Zaki, 2013; Nolan and Zaki, 2013) and fully-turbulent flows (Lee et al., 2013).

The parameters of the main simulations are summarized in Table 1. In the heated cases, the wall temperature was set to the desired value immediately downstream of the inlet. The fluid viscosity at the heated wall is 49.7% and 35.2% of the free-steam value in the MH and SH cases, respectively. The computational domain is a rectangular region with dimensions $L_x = 400\theta_{in}$, $L_y = 60\theta_{in}$ and $L_z = 80\theta_{in}$. The number of grid points is $4097 \times 385 \times 1281$ in x, y, and z, respectively. A non-uniform grid distribution was adopted in the wall-normal direction, whereas uniform grid spacing was used in the streamwise and spanwise directions. The grid spacing in the present study is summarized in Table 2 using wall units, and in Table 3 based on the Batchelor scale. Starting from the inlet Reynolds number $Re_\theta = 1240$, the reference unheated flow reaches $Re_\theta = 2060$ at the end of the streamwise domain. The total time for statistical averaging during the DNS is $1800\theta_{in}/U_\infty$ time units.

Due to the viscosity variation with temperature, an effective Reynolds number Re_θ^{eff} is defined:

$$Re_\theta^{eff} = \frac{U_\infty \theta}{\nu^{eff}}, \tag{4}$$

where

$$\nu^{eff} = \frac{1}{\delta} \int_0^\delta \bar{\nu}(y) dy. \tag{5}$$

All results are compared at the same Re_θ^{eff}. Because an appropriate inner length-scale is required in the presence of the non-uniform fluid viscosity, the ratio of the local viscosity to the friction velocity is used, i.e. $l_\nu(y) = \bar{\nu}(y)/u_\tau$, where the friction velocity u_τ is defined using the viscosity at the wall. Therefore, the modified inner scaling is given by $y_\nu^+ \equiv y/l_\nu(y)$.

Table 1
Summary of simulation parameters. The quantities $\nu_R|_w$ and $Pr(y)$ are determined at $Re_\theta^eff = 1840$. Note that the unheated (UH) case by Lee et al. (2013) corresponds to constant viscosity.

	T_w (°C)	T_∞ (°C)	$\nu_R\|_w$	$Pr(y)$	Δt (Θ_{in}/U_∞)
Constant viscosity (UH)	–	–	1.000	5.4	0.025
Moderately heated (MH)	70	30	0.497	2.68–5.4	0.018
Strongly heated (SH)	99	30	0.352	1.90–5.4	0.015

Table 2
Spatial and temporal resolutions normalized by wall units.

	Δx^+	Δy^+_{min}	Δy^+_{max}	Δz^+	Δt^+
Constant viscosity (UH)	5.08	0.246	24.6	3.25	0.0564
Moderately heated (MH)	9.22	0.447	22.4	5.90	0.0670
Strongly heated (SH)	12.2	0.593	21.1	7.82	0.0698

Table 3
Spatial and temporal resolutions normalized by the Batchelor scale ($\eta_\Theta = \eta\sqrt{1/Pr}$). Subscript δ denotes the value at the free-stream edge of the momentum boundary layer.

	$(\Delta x/\eta_\Theta)_{max}$	$(\Delta y/\eta_\Theta)_{max,w}$	$(\Delta y/\eta_\Theta)_{max,\delta}$	$(\Delta z/\eta_\Theta)_{max}$
Constant viscosity (UH)	8.3	0.402	3.04	5.31
Moderately heated (MH)	11.0	0.531	3.01	7.01
Strongly heated (SH)	12.4	0.599	2.98	7.91

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