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Influences of boundary conditions on laminar natural convection in rectangular enclosures with differentially heated side walls

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ABSTRACT

In this study, two-dimensional steady-state simulations of laminar natural convection of Newtonian fluids in rectangular enclosures with differentially heated side walls have been conducted. Two Prandtl numbers $Pr = 0.71$ and 7.0 – typical values for air and water – and a range of different aspect ratios $AR(=H/L$ where H is the enclosure depth and L is the enclosure width) ranging from 1/8 to 8 for constant heat flux boundary conditions are investigated for Rayleigh numbers in the range 10^4–10^6. To demonstrate the difference between the aspect ratio effects between the constant wall temperature and constant wall heat flux boundary conditions, simulations have also been carried out for the same range of numerical values of Rayleigh number for the constant wall temperature boundary condition. It is found that the mean Nusselt number $\overline{Nu}$ increases with increasing values of Rayleigh number for both constant wall temperature and constant heat flux boundary conditions. The effects of aspect ratio AR have also been investigated in detail and it has been found that the effects of thermal convection (diffusion) strengthens (weakens) with increasing aspect ratio and *vice versa*, for a given set of nominal values of Rayleigh number and Prandtl number for both types of boundary conditions. In the case of constant wall temperature boundary condition, the mean Nusselt number increases up to a certain value of the aspect ratio AR_{max} but for $AR > AR_{max}$ the mean Nusselt number starts to decrease with increasing AR. In contrast, the mean Nusselt number is found to increase monotonically with increasing AR for the constant wall heat flux boundary condition in the range of values of aspect ratio, Rayleigh number and Prandtl number considered in this study. Detailed physical explanations are provided for the observed phenomenon. Suitable correlations are proposed for the mean Nusselt number $\overline{Nu}$ for both constant wall temperature and wall heat flux boundary conditions which are shown to satisfactorily capture the correct qualitative and quantitative behaviour of $\overline{Nu}$ for the range of Rayleigh number and aspect ratio considered here.

1. Introduction

Natural convection in rectangular enclosures filled with Newtonian fluids has been analysed extensively by several researchers and interested readers are referred to Ostrach (1988), Gebhart et al. (1988), Khalifa (2001) and Ganguli et al. (2009) for detailed reviews. Although several boundary conditions for this problem are possible, differentially heated vertical side walls is one of the most analysed configurations (e.g. deVahl Davis, 1983). Unless stated otherwise, to avoid unnecessary repetition, the remainder of this paper will deal with this configuration for different aspect ratios (i.e. $AR = H/L$, in which H is the enclosure depth and L is the enclosure width) for Newtonian fluids. It has been shown in several previous studies (e.g. Elder, 1965; Gill, 1966; Newell and Schmidt, 1970; Yin et al., 1978; Bejan, 1979; Elsherbiny et al., 1982; Lee and Korpela, 1983; Wakitani, 1996) that AR plays a major role in the natural convection process in this configuration. However, natural convection in rectangular enclosures is often studied separately for tall ($AR \gg 1$) and shallow ($AR \ll 1$) enclosures and the vertical side walls are usually subjected to constant temperatures. Based on an extensive experimental analysis of this problem for tall enclosures Elder (1965) identified three distinct regions; a region in the vicinity of the vertical side walls where the temperature gradients are nearly horizontal and largest, an interior region where vertical temperature gradients appear and an end region strongly influenced by the boundary conditions. For small values of Rayleigh number[1] (i.e. $Ra_{CWT} < 10^3$), the isotherms remain parallel to the

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[1] The definition of Rayleigh number Ra_{CWT} will be provided later in Section 2 of this paper.

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Nomenclature

a constant (–)
AR aspect ratio ($AR = H/L$) (–)
c_A, c_B, c_C correlation parameter (–)
c_p specific heat at constant pressure (J/kg K)
c_1, c_2, c_3 correlation parameter (–)
C_B correlation parameter (–)
e relative error (–)
f_1, f_2, f_3, f_4 functions relating thermal and hydrodynamic boundary layers (–)
F fraction determining the ratio of the hydrodynamic boundary layer thickness on horizontal surface to the height of the enclosure (–)
g gravitational acceleration (m/s^2)
g_1, g_2 functions (–)
Gr_{CWT} Grashof number for the constant wall temperature configuration (–)
Gr_{CWHF} Grashof number for the constant wall heat flux configuration (–)
h heat transfer coefficient (W/m^2 K)
H height of the enclosure (m)
k thermal conductivity (W/m K)
K thermal gradient in horizontal direction (K/m)
L length of the enclosure (m)
n correlation parameter (–)
n_B (–) exponent of aspect ratio for self similar variation of mean Nusselt number in the boundary-layer regime (–)
Nu Nusselt number (–)
Nu_1 convective contribution to Nusselt number (–)
Nu_2 conduction contribution to Nusselt number (–)

Subscripts
c core
cen centre of the domain
$conv$ convection contribution
C cold wall
CWHF constant wall heat flux
CWT constant wall temperature
$diff$ diffusion contribution
ext extrapolated value
H hot wall

Special characters
T^* reference temperature ($=T_C$ for CWT case and $=T_{cen}$ for CWHF case) (K)
ΔT difference between hot and cold wall temperature ($=(T_H - T_C)$) (K)
ΔT_1 the temperature difference between the horizontal walls (K)
ΔT_{char} characteristic temperature difference (K)
ΔT_{cond} temperature difference between vertical walls for pure conduction (K)
$\overline{Nu}$ mean Nusselt number (–)
p apparent order of accuracy (–)
P pressure (N/m^2)
Pr Prandtl number (–)
q wall heat flux (W/m^2)
q_e, q_f correlation parameters (–)
Q thermal energy flow rate evaluated using an energy-flux integral over any cross-section at a given height (W)
Ra_{CWT} Rayleigh number for the constant wall temperature configuration (–)
Ra_{CWHF} Rayleigh number for the constant wall heat flux configuration (–)
T temperature (K)
u_i ith velocity component (m/s)
U, V dimensionless horizontal ($U = u_1L/\alpha$) and vertical velocity ($V = u_2L/\alpha$) (–)
ϑ characteristic velocity in vertical direction (m/s)
x_i coordinate in ith direction (m)
α thermal diffusivity (m^2/s)
β coefficient of thermal expansion (1/K)
δ_{ij} Kronecker's delta (–)
δ, δ_{th} hydrodynamic and thermal boundary layer thickness (m)
θ_{CWT} dimensionless temperature, ($\theta_{CWT} = (T - T_{cen})/(T_H - T_C)$) (–)
θ_{CWHF} (–) dimensionless temperature ($\theta_{CWHF} = (T - T_{cen})k/qL$) (–)
μ dynamic viscosity (N s/m^2)
$\boldsymbol{\nu}$ kinematic viscosity (m^2/s)
ρ density (kg/m^3)
$\tau_{ij}(\tau)$ stress tensor (Shear stress) (Pa)
φ general primitive variable
ψ stream function (m^2/s)
max maximum value
ref reference value
$wall$ wall value
$\Delta_{\min,cell}$ minimum cell distance (m)
r_x, r_y grid expansion ratio in x_1 and x_2 directions (–)

vertical boundaries and heat transfer takes place primarily due to conduction. For $10^3 < Ra_{CWT} < 10^5$, large values of temperature gradients are confined to near wall regions and a temperature stratification with an almost uniform vertical temperature gradient is established at the core. Complex secondary and tertiary flows appear in the interior region for larger values of Ra_{CWT}. Gill (1966) and Newell and Schmidt (1970) used analytical and computational methods respectively, to analyse natural convection in tall enclosures for high values of Ra_{CWT} where most of the heat transfer takes place in the thermal boundary layer adjacent to the vertical side walls. Gill (1966) obtained an asymptotic solution for tall enclosures in the limit of very large values of Rayleigh number (i.e. $Ra_{CWT} \to \infty$) with finite values of AR by matching the conditions of the core flow with the top and bottom boundary layers. Bejan (1979) subsequently refined the analysis of Gill (1966) and proposed expressions for mean values of Nusselt number for tall enclosures which are in good agreement with experimental data (Elder, 1965). A number of papers have investigated natural convection in tall enclosures both experimentally (Yin et al., 1978; Elsherbiny et al., 1982; Wakitani, 1996) and numerically (Lee and Korpela, 1983; Le Quéré, 1990; Wakitani, 1997; Zhao et al., 1997; Frederick, 1999; Lartigue et al., 2000; Dong and Zhai, 2007; Ganguli et al., 2009) and interested readers are referred to Ganguli et al. (2009) and references therein for a more detailed discussion.

Cormack et al. (1974a) analytically analysed natural convection in shallow enclosures (i.e. $AR \ll 1$) under asymptotic conditions of $AR \to 0$ for constant values of Ra_{CWT} and identified two convection regimes, namely, the parallel-flow regime and boundary layer regime. In the parallel flow regime, two horizontal counter-currents appear in the central core and the horizontal temperature gradient remains uniform throughout the interior region with the isotherms parallel to the vertical walls. In contrast, in the boundary-layer regime the regions of high thermal gradients are confined in the boundary layers adjacent to the vertical walls and convection