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Turbulent natural convection scaling in a vertical channel

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ABSTRACT

Using direct numerical simulation (DNS) data, this study appraises existing scaling laws in literature for turbulent natural convection of air in a differentially heated vertical channel. The present data is validated using past DNS studies, and covers a range of Rayleigh number, *Ra* between 5.4×10^5 and 2.0×10^7. We then appraise and compare the various scaling laws proposed by Versteegh and Nieuwstadt (1999), Hölling and Herwig (2005), Shiri and George (2008) and George and Capp (1979) with the profiles of the mean temperature defect, mean streamwise velocity, normal velocity fluctuations, temperature fluctuations and Reynolds shear stress. Based on the arguments of an inner (near-wall) and outer (channel centre) region, the data is found to support a minus one-third power law for the mean temperature in an overlap region. Using the inner and outer temperature profiles, an implicit heat transfer equation is obtained and we show that a *correction term* is non-negligible for the present *Ra* range when compared with explicit equations found in literature. In addition, we determined that the mean streamwise velocity and normal velocity fluctuations collapse in the inner region when using the outer velocity scale. We also find that the temperature fluctuations scale in inner coordinates, in contrast to the outer scaling behaviour reported in the past. Lastly, we show evidence of an incipient proportional relationship between friction velocity, u_τ, and the outer velocity scale, u_o, with increasing *Ra*.

1. Introduction

In the building industry, it is necessary for ventilation-design engineers to understand the control of heating, ventilation and air-conditioning parameters of occupied spaces, because, any form of control influences the design of buildings. One of the prevalent industry practices is to control the heat gain or loss of the building by insulating the external building envelope (Batchelor, 1954). This can take various forms such as double-paned windows, insulated cavity in external walls and double-skin façades. In general, we find that for such forms, the air flow is naturally-convected and wall-bounded. So, to study the insulating nature of these flows, the present study aims to appraise the scaling arguments and wall functions (or wall models) in existing literature and in light of newer, higher-resolution direct numerical simulation (DNS) data.

For computational fluid dynamics (CFD) simulations such as Reynolds-averaged Navier–Stokes simulations (RANS) and large-eddy simulations (LES), it is important to have accurate wall models, especially since these models set the boundary conditions for calculations close to the wall. These models reduce the added computational cost associated with fine grid spacings, a requirement for capturing rapid changes in flow physics near the wall (Kiš and Herwig, 2012). As such, it is important to have accurate wall models that are valid for a large range of flow conditions.

Many studies in the past have developed wall functions using various scaling analyses (e.g. George and Capp, 1979; Yuan et al., 1993; Hölling and Herwig, 2005; Shiri and George, 2008). These wall functions are typically formed with unknown constants, and then fitted to experimental and numerical data (e.g. Versteegh and Nieuwstadt, 1999; Henkes and Hoogendoorn, 1990). However, the opinions of these studies have largely been found to differ, possibly due to the limited numerical data at the time.

This research focuses on validating the aforementioned scaling analyses and wall functions with DNS data for Rayleigh numbers (*Ra*) up to 2.0×10^7. To date, the DNS data by Versteegh and Nieuwstadt (1999) for *Ra* up to 5.0×10^6 has been most frequently cited, while the more recent study by Kiš and Herwig (2012) of *Ra* up to 2.3×10^7 is the highest-known DNS data available for comparison. From the DNS data, Versteegh and Nieuwstadt (1999) proposed constants for their wall functions based on empirical fitting of their theoretical arguments to the data. However, Shiri and George (2008) questions the validity of the existing data at that time for appraising asymptotic theories and argues that, in order for these theories to be evaluated, the data should originate from a flow with a ratio of outer to inner length scales, h/l_i, greater than 10. This criterion is used throughout our simulations, and have been calculated to range between 19 and 62.

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2. Background

2.1. Governing equations

In the present study, we adopt the Boussinesq approximation (constant fluid properties except buoyancy, which is a function of temperature) for fully developed turbulent natural convection driven by temperature difference between the two walls, ΔT. Here, $\Delta T = T_h - T_c$, where T_h and T_c are defined as the temperatures of the hot and cold walls respectively. For all simulations in this study, the hot wall is located at the left wall (see Fig. 1) and the reference temperature, $T_{\rm ref} = (T_h - T_c)/2$, is defined as the average temperature across the channel. The relevant measure of the flow, analogous to the Reynolds number, is the Rayleigh number, Ra, defined as $Ra = g\beta\Delta T(2h)^3/(\nu\alpha)$, where g is the acceleration of gravity and ν, α and β are the kinematic viscosity, thermal diffusivity and thermal expansion coefficient of the fluid. The streamwise, spanwise and wall-normal directions are x, y and z respectively, and the channel width is $2h$. The Reynolds-averaged mean equations of motion can be written as:

$$0 = \frac{d}{dz}\left(\nu\frac{d\bar{u}}{dz} - \overline{u'w'}\right) + g\beta(\overline{T} - T_{\rm ref}), \quad (2.1)$$

$$0 = \frac{d}{dz}\left(\alpha\frac{d\overline{T}}{dz} - \overline{w'T'}\right), \quad (2.2)$$

(cf. George and Capp, 1979) where the mean flow is averaged over time as well as the streamwise and spanwise directions. The overbar denotes the average of the quantities while the fluctuating quantities are denoted by a prime. The Reynolds shear stress is $-\overline{u'w'}$ and the turbulent heat flux is $-\overline{w'T'}$.

2.2. Simulation parameters

The governing equations are solved numerically over a computational domain size defined by $L_x \times L_y \times L_z$, with the resolutions $n_x \times n_y \times n_z = 432 \times 216 \times 96$ for Ra up to 5.0×10^6—similar to the parameters used by Versteegh and Nieuwstadt (1999)—and $n_x \times n_y \times n_z = 768 \times 384 \times 192$ for $Ra = 2.0 \times 10^7$. Through our simulations, we have determined that the ratio of grid spacings to inner length scale, Δ/l_i of O(1) is sufficient to resolve the small scales. With exception of the highest Ra, we ensured that the ratio $t_{\rm sim}/t_{\rm eddy} > 100$, where $t_{\rm sim}$ is the length of time used to record statistics, and $t_{\rm eddy} = h/(g\beta f_w h)^{1/3}$ is the eddy turnover time. Table 1 lists the simulation parameters in this study.

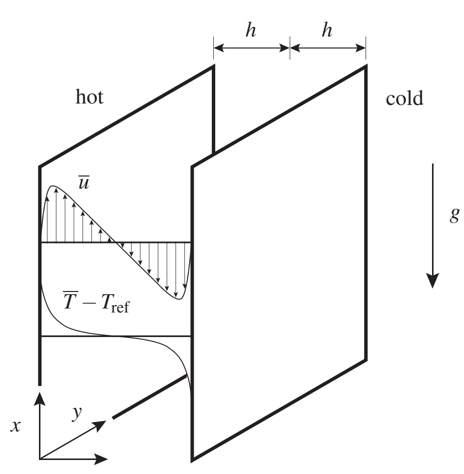

Fig. 1. Schematic diagram of natural convection in a vertical channel.

The governing equations are spatially discretised using the fully conservative fourth-order staggered scheme of Morinishi et al. (1998) and marched in time using the low-storage third-order Runge–Kutta scheme of Spalart et al. (1991). The velocity field is projected onto a divergence-free field after each Runge–Kutta stage via the fractional-step method (e.g. Kim and Moin, 1985). Grid spacings in the streamwise and spanwise directions are uniform, and the wall-normal spacings utilise a cosine stretching function, defined by

$$\Delta z(i_z) = \frac{L_z}{2}\left\{\cos\left[\frac{\pi(i_z - 1)}{n_z}\right] - \cos\left(\frac{\pi i_z}{n_z}\right)\right\} \quad (2.3)$$

where $\Delta z(i_z)$ is the wall-normal spacing and $i_z \in [1, n_z]$.

3. Comparison with published DNS data

To validate the present simulation, we compared the statistics with the data of Versteegh and Nieuwstadt (1999) and Pallares et al. (2010) for $Ra = 5.4 \times 10^5$. For brevity, we report only the turbulent statistics, and this is shown respectively in Fig. 2a and b. Overall, we found that our present data is consistent with published DNS data: the temperature fluctuation profiles in Fig. 2b

Table 1
Simulation parameters for this study. The cell gridsizes, $\Delta x^\times$, $\Delta y^\times$ and $\Delta z_c^\times$ are scaled by the inner length scale, $l_i = [\alpha^3/(g\beta|f_w|)]^{1/4}$. The wall normal gridsize $\Delta z_c^\times$ is measured at the channel centre.

Ra	L_x/h	L_y/h	$\Delta x^\times$	$\Delta y^\times$	$\Delta z_c^\times$	$t_{\rm sim}/t_{\rm eddy}$
5.4×10^5	24	12	1.1	1.1	0.6	192.7
2.0×10^6	24	12	1.6	1.6	1.0	147.0
5.0×10^6	24	12	2.2	2.2	1.3	117.4
2.0×10^7	24	12	1.9	1.9	1.22	51.5

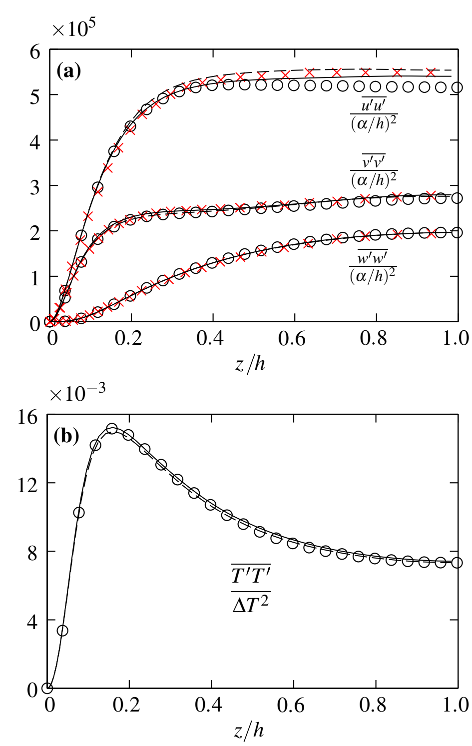

Fig. 2. DNS velocity and temperature fluctuation data for $Ra = 5.4 \times 10^5$ by Versteegh and Nieuwstadt (1999) (○) and Pallares et al. (2010) (×) compared with the present fourth-order (—) and second-order data (– –).