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Heat and mass transfer on a MHD third grade fluid with partial slip flow past an infinite vertical insulated porous plate in a porous medium

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ABSTRACT

The influence of third grade, partial slip and other thermophysical parameters on the steady flow, heat and mass transfer of viscoelastic third grade fluid past an infinite vertical insulated plate subject to suction across the boundary layer has been investigated. The space occupying the fluid is porous. The momentum equation is characterized by a highly nonlinear boundary value problem in which the order of the differential equation exceeds the number of available boundary conditions. An efficient numerical scheme of midpoint technique with Richardson's extrapolation is employed to solve the governing system of coupled nonlinear equations of momentum, energy and concentration. Numerical calculations were carried out for different values of various interesting non-dimensional quantities in the slip flow regime with heat and mass transfer and were shown with the aid of figures. The values of the wall shear stress, the local rate of heat and mass transfers were obtained and tabulated. The analysis shows that as the fluid becomes more shear thickening, the momentum boundary layer decreases but the thermal boundary layer increases; the magnetic field strength is found to decrease with an increasing temperature distribution when the porous plate is insulated. The consequences of increasing the permeability parameter and Schmidt number decrease both the momentum and concentration boundary layer thicknesses respectively whereas an increase in the thermal Grashof number gives rise to the thermal boundary layer thickness.

1. Introduction

The boundary layer flow of non-Newtonian fluids has been widely recognized by a number of investigators in view of their immense technological and scientific applications such as polymer production, food processing, oil recovery, manufacturing of ceramics or glasswork. In these fluids, the constitutive relationship between the stress and rate of strain is much complicated in comparison to the Navier–Stoke's equation. A subclass of differential type of the fluids; namely the second grade fluid has been reported much in the literature. This subclass can predict the normal stress differences even in the steady flow over a rigid boundary. However, a third grade fluid is required in order to investigate the shear-thinning/thickening effects. This fluid model has been scarcely studied despite its complex constitutive relationship in comparison to viscous and second grade fluids.

One can refer to the useful works of Chang Man Fong et al. (1996), Ariel (1994, 2007) regarding the flow of non-Newtonian fluid past an infinite plate. Mansutti et al. (1993) numerically studied the steady flow of three different classes of non-linear fluids of differential type (power-law, second and third grade fluids) past a porous plate with uniform suction or injection and compared solutions obtained by finite difference method versus those obtained by regular and singular perturbation methods in order to understand the effect of the augmented boundary conditions. Das et al. (2010) also investigated the hydromagnetic convective flow past a vertical porous plate through a porous medium with suction and heat source. Olajuwon (2011) investigated the convection heat and mass transfer in a hydromagnetic flow of a second grade fluid in the presence of thermal radiation and thermal diffusion. Numerical approach of Runge–Kutta shooting method was used to solve the problem and he presented the velocity, temperature and concentration fields for different values of parameters entering into the problem.

In many practical applications, the particle adjacent to a solid surface no longer takes the velocity of the surface. The particle has a finite tangential velocity; it 'slips' along the surface. The flow is called slip-flow and this effect cannot be neglected. One can cite the works of Andersson (2002), Hayat et al. (2009), Sahoo (2010a,b,c), Sahoo and Do (2010) and all the references therein

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Nomenclature

Alphabetical symbols

A_1, A_2, A_3	Rivlin–Ericksen tensor
B	magnetic field
B_0	magnetic strength intensity
C	ambient concentration
c_p	specific heat at constant pressure
C_w	concentration at the wall/plate
C_∞	free stream concentration
D	coefficient of mass diffusivity
Ec	Eckert number
Gc	solutal Grashof number
Gt	thermal Grashof number
g	dimensionless temperature
g'	acceleration due to gravity
H	magnetic field parameter
h_∞	terminal mesh spacing
I	identity tensor
i, j	horizontal and vertical component of V
J	electric current
K	normal stress parameter
m	permeability parameter
Pr	Prandtl number
p_1	pressure
Sc	Schmidt number
T	ambient temperature
T_w	temperature at the wall/plate
u_∞	free stream velocity
u, v	velocity components in x and y directions
V	ambient velocity field
V_0	suction or blowing
η_1, η_2	numerical constants
x, y	Cartesian coordinate
y_∞	terminal mesh point

Greek symbols

α_1, α_2	normal stresses
β	dimensionless third grade parameter
$\beta_1, \beta_2, \beta_3$	material moduli
β_C	volumetric coefficient of concentration expansion
β_T	volumetric coefficient of thermal expansion
δ	permeability of the medium
κ	thermal conductivity
λ	dimensionless partial slip parameter
μ	dynamic viscosity
ρ	fluid density
τ	Cauchy stress tensor
ϕ	dimensionless concentration
φ	porosity

Symbols

$\frac{d}{dt}$	material time derivative
∇	gradient operator
$\varDelta$	change operator
Δh	step size

Subscript and superscript

w	surface condition
p	constant pressure
∞	condition at the free stream

regarding the flow and heat transfer of Newtonian and non-Newtonian fluids with partial slips and diverse physical effects. Abelman et al. (2009) investigated the Couette flow a third grade fluids with rotating frame and slip condition which occupies the porous space between two rigid infinite plates. They obtained a numerical solution using MATLAB. Ellahi et al. (2010) examined the heat transfer analysis on the laminar flow of an incompressible third grade fluid through a porous flat channel. They compared the analytical solution obtained for temperature distribution for various values of the controlling parameters with the numerical solution.

Ellahi et al. (2012) also presented the effects of heat and mass transfer with slip on the Couette and generalized Couette flow in a homogeneous and thermodynamically compatible third grade non-Newtonian viscous fluid. They derived exact solutions of velocity and temperature in Couette flow problem and the nonlinear analysis for generalized Couette flow problem was performed by using spectral homotopy analysis method (SHAM). Ellahi and Hameed (2012) further numerically investigated the steady non-Newtonian flows with heat transfer, MHD and nonslip effects. Ashgar et al. (2007) studied the influence of a partial slip on flows of a second grade fluid in a porous medium using Fourier transform. They concluded that the velocity profile was of the wave nature and that amplitude of the wave decreases when the partial slip parameter increases. Khan et al. (2013) reported the effects of variable viscosity on the peristaltic motion of an incompressible non-Newtonian fluid with variable viscosity through a porous medium in an inclined symmetric channel with slip boundary conditions. They employed regular perturbation method to solve the system of the governing nonlinear partial differential equations.

The study of magnetohydrodynamic flow, heat and mass transfer of a viscoelastic third grade fluid with partial slip in a porous medium has not received much attention in the literature. In this paper, we present a numerical solution for a magnetohydrodynamic flow with partial slip, heat and mass transfer of a thermodynamically compatible third grade fluid over a vertical insulated porous plate embedded in a porous medium.

2. Mathematical analysis

Consider the viscometric flow, heat and mass transfer of an incompressible fluid of third grade past a semi-infinite non-conducting porous plate subject to a transverse, uniform magnetic field $B = (0, B_0, 0)$ in a porous medium. The surface of the plate is insulated and admits partial slip condition. The leading edge of the plate is at $x = 0$. The x-coordinate is measured from the leading edge of the plate and the plate is coinciding with the plane $y = 0$. The corresponding components of the velocity V in the x and y directions are u and v respectively. The plate temperature is at temperature T_w and the concentration of the fluid at the surface is C_w. The flow far away from the plate is uniform and is in the direction parallel to the plate. The temperature and concentration of the fluid far away from the plate are T_∞ and C_∞ respectively. Thus, for the problem under consideration, we respectively seek velocity, temperature and concentration field of the forms:

$$V = u(y)i + v(y)j \tag{2.1}$$

$$T = T(y) \tag{2.2}$$

$$C = C(y) \tag{2.3}$$

where T denotes the ambient temperature and C stands for the ambient species concentration. The fluid is magnetohydrodynamic with small magnetic Reynold's number so that the induced