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Stability analysis of soret-driven double-diffusive convection of Maxwell fluid in a porous medium

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ABSTRACT

Stability analysis of double-diffusive convection for viscoelastic fluid with Soret effect in a porous medium is investigated using a modified-Maxwell–Darcy model. We use the linear stability analysis to investigate how the Soret parameter and the relaxation time of viscoelastic fluid effect the onset of convection and the selection of an unstable wavenumber. It is found that the Soret effect is to destabilize the system for oscillatory convection. The relaxation time also enhances the instability of the system. The effects of Soret coefficient and relaxation time on the heat transfer rate in a porous medium are studied using the nonlinear stability analysis, the variation of the Nusselt number with respect to the Rayleigh number is derived for stationary and oscillatory convection modes. Some previous results can be reduced as the special cases of the present paper.

1. Introduction

Double-diffusive convection in porous media due to temperature and convection and concentration gradients has been widely studied because of its numerous fundamental and industrial applications. The double-diffusive convection is of importance in various fields such as high quality crystal production, liquid gas storage, oceanography, production of pure medication, solidification of molten alloys, and geothermally heated lakes and magmas. The enormous volume of work devoted to this field is well documented in the books by Nield and Bejan (1999), Ingham and Pop (2002), Vafai (2000), Pop and Ingham (2001). However, in a binary fluid, the cross-coupling between thermal diffusion and solutal diffusion should not be negligible. The Swiss scientist, Charles Soret, discovered that a salt solution contained in a tube with the two ends at different temperatures did not remain uniform in composition in 1879 (Soret, 1879). The salt was more concentrated near the cold end than near the hot end of the tube. The phenomenon of a flux of solute generated by a temperature gradient is known as the Soret effect. During the past decades, the study of Soret effect on a developed natural convection in porous media has received great importance and interests (Bergeron et al., 1998; Bourich et al., 2002, 2004; Mahidjiba et al., 2006; Malashetty et al., 2006).

Charrier-Mojtabi et al. (2007) studied the linear and nonlinear stability of the equilibrium solution and the monocellular flow in a horizontal porous layer filled by a binary fluid and heated from below or above. The Soret effect is taken into account and the influence of both the separation ratio and the normalized porosity is studied. Bennacer et al. (2003) considered the Soret effect on convection in a horizontal porous cavity submitted to cross gradients of temperature and concentration. Their results showed that, when the vertical concentration gradient is stabilizing, multiple steady-state solutions become possible over a range of buoyancy ratios which is strongly dependent on the Soret parameter. Bahloul et al. (2003) considered the double-diffusive and Soret-induced convection in a shallow horizontal porous layer, and the stability of the parallel flow solution is studied, then the threshold for Hopf bifurcation is determined. Bourich et al. (2005) studied analytically and numerically the Soret effect on thermal natural convection within a horizontal porous enclosure uniformly heated from below by a constant heat flux using the Brinkman-extended Darcy model. It is found that the separation parameter has a strong effect on the thresholds of instabilities and the heat and mass transfer characteristics.

Recently, interest in viscoelastic flows through porous media has also grown considerably, due largely to the demands of such diverse areas as biorheology, geophysics, chemical and petroleum industries (Capuani et al., 2003; Hayat et al., 2007; Khaled and Vafai, 2003; Masuoka et al., 2003; Wang, 2002; Younes, 2003). The mathematical model of Maxwell fluid has been served as a simplified description of dilute polymeric solutions/fluids (Raikher

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and Rusakov, 1996; Speziale, 2000) and has also been used to describe the rheology of flour doughs (Schofield and Scott Blair, 1932), as well as other viscoelastic fluids, include glycerin (Raman and Venkateswaran, 1939), toluene (Reiner, 1960), crude oil (Tsiklauri and Beresnev, 2001a, 2001 b), etc. Based on the thermodynamical analysis and using symmetry and frame indifferent arguments, Preziosi and Farina found out that the Darcy model needs to be modified to account for the mass exchange (Preziosi and Farina, 2002). Making an analogy with the constitutive equation of the Maxwell fluid, the following phenomenlogical model for non-Newtonian fluids in porous medium is introduced (Khuzhayorov et al., 2000; Tan and Masuoka, 2007)

$$\left(1+\bar{\lambda}\frac{\partial}{\partial t}\right)\nabla p=-\frac{\mu}{K}\mathbf{V}, \tag{1}$$

which is called the modified-Darcy–Maxwell model, where $\bar{\lambda}$ is the stress relaxation characteristic time constant, K the permeability of the porous medium, μ the effective fluid viscosity in the porous medium, p the pressure, and $\mathbf{V}$ the Darcian velocity. Comparison Eq. (1) with the constitutive equation of Maxwell fluid, it is obviously that the time derivative in Eq. (1) is related to the non-Newtonian behavior of real fluids in porous medium.

However, the study of Soret-driven double-diffusion convection of viscoelastic fluid in a porous medium has not been given any attention in spite of its importance in many practical applications. In the present study, the modified-Darcy–Maxwell model is considered for a porous medium saturated by the viscoelastic fluid. The influences of the Soret effect and relaxation time on the natural convective flows between two parallel infinite stress-free boundaries and heated from below by a constant temperature were studied. The effects of Soret coefficient and relaxation time on the Nusselt number is studied, respectively.

2. Basic equations

We consider a system consisting of a homogeneous and isotropic horizontal porous layer saturated with Boussinesq Maxwell fluid confined between parallel boundaries $z = 0$ and $z = d$, which are maintained at constant but different temperatures and solute concentrations T_1, S_1 and T_2, S_2 ($T_1 > T_2$, $S_1 > S_2$), respectively. Then the onset of double-diffusive convection can be studied under the Boussinesq approximation and an assumption that the density of the fluid ρ depends linearly on the temperature T and solute concentration S

$$\rho_f=\rho_0[1-\beta_T(T-T_0)+\beta_S(S-S_0)], \tag{2}$$

where ρ_f and ρ_0 are the densities at the current and reference state, respectively. The quantities β_T and β_S are the coefficients for thermal and solutal expansion, respectively. The subscript 0 denotes the reference state. Because of the Boussinesq approximation, which states that the effect of compressibility is negligible everywhere in the conservations except in the buoyancy term, is assumed to hold, then the equations for conservation of mass, momentum read, respectively

$$\nabla\cdot\mathbf{V}=0, \tag{3}$$

$$\frac{\mu}{K}\mathbf{V}=\left(1+\bar{\lambda}\frac{\partial}{\partial t}\right)(-\nabla p+\rho\mathbf{g}), \tag{4}$$

where $\mathbf{V} = (u, v, w)$ is the volume average velocity obtained by the local volume averaging technique (Ingham and Pop, 2002; Slattery, 1999) and $\mathbf{g}$ is the acceleration due to gravity.

The phenomenological equations relating the fluxes of heat $\mathbf{J}_T$ and matter $\mathbf{J}_C$ to the thermal and solute gradients present in a binary fluid mixture are given by De Groot and Marur (1962)

$$\mathbf{J}_T=-k\nabla T \tag{5}$$

$$\mathbf{J}_C=-\rho D\nabla S-\rho D' S(1-S)\nabla T \tag{6}$$

where k and D are the thermal conductivity and the mass diffusivity of species through the fluid-saturated porous medium, respectively, D' is the effect thermal diffusion coefficient.

Then the equations expressing conservation of energy and species are given by

$$(\rho C)_p\frac{\partial T}{\partial t}+(\rho C)_f(\mathbf{V}\cdot\nabla)T=k\nabla^2 T \tag{7}$$

$$\phi\frac{\partial S}{\partial t}+(\mathbf{V}\cdot\nabla)S=D\nabla^2 S+D'S(1-S)\nabla^2 T \tag{8}$$

where $(\rho C)_p$ and $(\rho C)_f$ are respectively the heat capacity of the fluid and the saturated porous medium. Hurle and Jakeman (1971) treat the term $S(1-S)$ in the Soret effect as having a mean constant value, hence we here treat the Soret term to have a constant coefficient. From the mathematical viewpoint we may write the concentration equation, in the presence of Soret effect in the following form:

$$\phi\frac{\partial S}{\partial t}+(\mathbf{V}\cdot\nabla)S=D\nabla^2 S+D_s\nabla^2 T, \tag{9}$$

here D_s quantifies the contribution to the mass flux due to temperature gradient.

The basic state of the fluid is quiescent and is given by

$$\begin{array}{l}\mathbf{V}_b=(0,0,0)\quad P=P_b(z),\quad \rho=\rho_b(z),\quad T=T_b(z)\\ S=S_b(z),\quad \frac{dP_b}{dz}=\rho_b g,\quad \frac{d^2T_b}{dz^2}=0,\quad \frac{d^2S_b}{dz^2}=0\end{array}. \tag{10}$$

Then we superpose perturbation on the basic state in the form

$$\begin{array}{l}\mathbf{V}=\mathbf{V}_b+\mathbf{V}',\quad P=P_b(z)+P'\\ \rho=\rho_b(z)+\rho',\quad T=T_b(z)+T',\quad S=S_b(z)+S'\end{array}, \tag{11}$$

where the primes indicate perturbations. Then we have

$$\nabla\cdot\mathbf{V}'=0 \tag{12}$$

$$\frac{\mu}{K}\mathbf{V}'=-\left(1+\bar{\lambda}\frac{\partial}{\partial t}\right)\left[\nabla p'-\mathbf{k}g\rho_0(\beta_T T'-\beta_S S')\right] \tag{13}$$

$$(\rho C)_p\frac{\partial T'}{\partial t}+(\rho C)_f\left[(\mathbf{V}'\cdot\nabla)T'+w'\frac{\partial T_b}{\partial z}\right]=k\nabla^2 T' \tag{14}$$

$$\phi\frac{\partial S'}{\partial t}+(\mathbf{V}'\cdot\nabla)S'+w'\frac{\partial S_b}{\partial z}=D\nabla^2 S'+D_s\nabla^2 T' \tag{15}$$

We define the stream function ψ by $(u',w')=(\frac{\partial\psi}{\partial z},-\frac{\partial\psi}{\partial x})$, which satisfies the continuity equation. Eliminating the pressure term form Eq. (13), introducing the stream function ψ and non-dimensionalizing the resulting equation as well as Eqs. (14) and (15) using the following non-dimensional parameters:

$$(x^*,z^*)=\left(\frac{x}{d},\frac{z}{d}\right);\quad t^*=\frac{t\kappa}{d^2M},\quad \psi^*=\frac{\psi'}{\tau},\quad T^*=\frac{T'}{\Delta T},\quad S^*=\frac{S'}{\Delta S}, \tag{16}$$

here $\kappa = \tau/(\rho C)_f$ is the thermal diffusivity of the porous medium, $M = (\rho C)_p/(\rho C)_f$ is a dimensionless number which indicates the heat capacity ratio. Then we obtain (for simplicity, the asterisks, i.e., the dimensionless mark will be neglected hereinafter)

$$\nabla^2\psi=-\left(1+\lambda\frac{\partial}{\partial t}\right)\left(Ra\frac{\partial T}{\partial x}+Rs\frac{\partial S}{\partial x}\right) \tag{17}$$

$$\frac{\partial T}{\partial t}+\frac{\partial\psi}{\partial x}-\frac{\partial(\psi,T)}{\partial(x,z)}=\nabla^2 T \tag{18}$$

$$\varepsilon\frac{\partial S}{\partial t}+\frac{\partial\psi}{\partial x}-\frac{\partial(\psi,S)}{\partial(x,z)}=\frac{1}{Le}\left[\nabla^2 S+Sr\nabla^2 T\right] \tag{19}$$

where $\lambda=\frac{\bar{\lambda}\kappa}{d^2M}$ is dimensionless relaxation time, $Ra=\frac{\beta_T g\triangle Td^3}{\nu\kappa}$ is thermal Rayleigh number, $Rs=\frac{\beta_S g\triangle Sd^3}{\nu\kappa}$ is solutal Rayleigh number,