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Heat transfer characteristics of developing gaseous slip-flow in rectangular microchannels with variable physical properties

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ABSTRACT

The effects of variable physical properties on the flow and heat transfer characteristics of simultaneously developing slip-flow in rectangular microchannels with constant wall temperature are numerically investigated. A colocated finite-volume method is used in order to solve the mass, momentum and energy equations in their most general form. Various channel aspect ratios are studied at different Knudsen numbers. Simulations indicate that the constant physical property assumption can result in under/over-prediction of the local friction and heat transfer coefficients depending on the problem configuration. Density and thermo-physical property variations have significant effects on predicting flow and heat transfer characteristics in the developing and fully-developed regions. The degree of discrepancy varies for different cases depending on Knudsen number, aspect ratio and the temperature difference between the channel inlet and the wall. The results suggest that even low temperature differences can alter the friction and heat transfer coefficients considerably.

1. Introduction

Micro devices have gained significant popularity in recent years due to their applicability in various fields from cooling devices to drug delivery systems. As these devices become more and more complicated, there is a push towards smaller dimensions where the conventional theories of macro-devices begin to fail. A good example is the slip-flow effect in microfluidic devices where the no-slip assumption at the walls breaks down, and in the case of heat or mass transfer, a temperature or concentration-jump is observed between the surface and the adjacent fluid layer. The Knudsen number, Kn, defined as the ratio of molecular mean-free-path to the characteristic length scale of the problem, can be used to estimate the applicability of the continuum equations in dealing with problems that involve very small length scales or rarefied gases. For finite values of the Knudsen number, the continuum equations cannot be applied directly and either they should be modified or molecular models should be employed. In the case of rarefied gas flow, it is known that for $Kn < 0.001$ the continuum models are valid, and for $Kn > 10$ the free-molecular models should be employed. In the mid range, neither continuum models nor free-molecular models are satisfactory and another classification is needed: slip-flow for the range $0.001 < Kn < 0.1$ and transition-flow for the range $0.1 < Kn < 10$ are considered to be appropriate descriptions (Kennard, 1938). In the slip-flow regime, the continuum equations can still be employed but proper velocity-slip and temperature-jump boundary conditions should be specified. Experimental, analytical and numerical studies have confirmed the applicability of the continuum equations along with proper slip/jump boundary conditions in the slip-flow range of Knudsen numbers (e.g., Beskok and Karniadakis, 1992; Harley et al., 1995; Arkilic et al., 1997; Morini and Spiga, 1998; Beskok, 1999; Meinhart et al., 1999; Turner et al., 2004; Morini et al., 2004; Hsieh et al., 2004).

Morini (2004) conducted a comprehensive review of convective heat transfer in microchannels. He concluded that besides the disagreement between the friction factor and Nusselt number in microchannels with conventional theory predictions, the reported flow and heat transfer characteristics of microchannels are inconsistent among different researchers. He attributed this discrepancy to a number of factors such as surface conditions, rarefaction and compressibility, property variation and viscous heating effects. Most of the analytical solutions and numerical simulations are limited to 2-D geometries with simplifying assumptions for different boundary conditions. Kavehpour et al. (1997) studied the effect of compressibility and slip-flow in planar microchannels with first order

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slip/jump boundary conditions. They reported reduced friction and heat transfer coefficients compared to no-slip continuum results. Chen (2004) numerically studied density variation effect on friction coefficient in long rectangular microchannels. Aydin and Avci (2006) investigated hydrodynamically developed but thermally developing rarefied gas flow between two parallel plates considering the viscous dissipation effect. They reported the presence of singularities in Nusselt number variation for the constant wall temperature case. Van Rij et al. (2007) analyzed the importance of second order slip/jump boundary conditions on simultaneously developing flow in a planar geometry with constant wall heat flux. Liu et al. (2008) studied the variable-property effect of thermally developing no-slip liquid flow in microchannels and reported higher heat transfer rates compared to constant-property simulations. A perturbation-based analytical solution for fully-developed slip-flow with constant wall temperature in 2D channels is presented by Hooman et al. (2009) and Hooman and Ejlali (2010). Their solution considers variable viscosity and thermal conductivity with viscous heating taken into account. Most recently, Gulhane and Mahulikar (2010) investigated the effect of property variations on convective heat transfer characteristics in micro tubes.

Flow and heat transfer properties of rectangular microchannels with slip/jump boundary conditions have also been studied. Reviewing the results of theoretical and experimental studies, Rostami et al. (2000) concluded that the available conventional macro-channel theories are not adequate to predict the flow and heat transfer characteristics of gaseous flow in microchannels. Morini and Spiga (1998) analytically determined the velocity field in fully-developed incompressible laminar slip-flow in rectangular microchannels of arbitrary aspect ratio. Yu and Ameel (2001) used an integral transform technique to solve the energy equation with no axial conduction assuming a fully-developed incompressible slip-flow field. Renksizbulut et al. (2006) studied the effect of rarefaction at the entrance region of rectangular microchannels numerically, for different channel aspect ratios with constant physical properties. Kuddusi and Ceten (2007), Kuddusi (2007) and Ghodoossi and Egrican (2005) conducted analytical studies on the heat transfer properties of thermally and hydrodynamically developed incompressible flows under different boundary condition combinations. More recently, Hettiarachchi et al. (2008) studied slip-flow in rectangular microchannels with a constant wall temperature boundary condition numerically with developing velocity and temperature fields. In their simulations, they neglected viscous dissipation and assumed constant physical properties. Husain and Kim (2008) used temperature dependent thermo-physical property simulations to optimize the design of rectangular microchannel heat sinks without velocity-slip or temperature jump at the wall. Nonino et al. (2006, 2007) studied the effect of temperature dependent viscosity on heat transfer characteristics of no-slip liquid flow in 2D and 3D channels. Their results suggest that the temperature dependence of viscosity cannot be ignored within a considerable range of working conditions especially at the channel inlet. Van Rij et al. (2009a,b) studied the effect of viscous dissipation and second order slip/jump boundary conditions on flow and heat transfer characteristics of rectangular microchannels. In order to avoid any considerable changes in physical properties, they chose a very small temperature difference between the inlet and the wall and employed constant physical properties in their simulations.

In the present work, three dimensional gaseous slip-flow and heat transfer in rectangular microchannels of different aspect ratios are studied numerically for developing flow and temperature fields at different Knudsen numbers. The governing equations are in their most general form with all terms included. Also, the compressibility effect and variation of physical properties with temperature are included in the simulations.

2. Mathematical formulation

A schematic view of the microchannel and the coordinate system is depicted in Fig. 1. The aspect ratio of the channel is $\alpha = W/H$ with H and W being the channel height and width, respectively. The flow direction is along the x axis. For the low Peclet number values used in this study, a channel length of $6D_h$ is sufficient for the flow and temperature field to develop (Van Rij et al., 2009a), with $D_h = 2HW/(H+W)$ being the hydraulic diameter of the channel. Also, since the main scope of the present work is to examine the importance of property variations (mainly due to temperature change) on the flow and heat transfer characteristic, a short microchannel is chosen for the simulations.

The steady-state variable-property conservation equations can be expressed in non-dimensional form as:

$$\frac{\partial}{\partial x_i}(\rho u_i) = 0 \tag{1}$$

$$\rho u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial}{\partial x_j}(p\delta_{ij}) + \frac{1}{\mathrm{Re}}\frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\mu\frac{\partial u_k}{\partial x_k}\delta_{ij}\right] \tag{2}$$

$$\begin{aligned}\rho c_p \tfrac{\partial}{\partial x_i}(u_i T) &= (\gamma_0 - 1)\mathrm{Ma}^2 \tfrac{\partial}{\partial x_i}(u_i p) + \tfrac{1}{\mathrm{RePr}}\tfrac{\partial}{\partial x_i}\left(k\tfrac{\partial T}{\partial x_i}\right) + \\ &\quad (\gamma_0 - 1)\tfrac{\mathrm{Ma}^2}{\mathrm{Re}}\left[\tfrac{1}{2}\mu\left(\tfrac{\partial u_i}{\partial x_j} + \tfrac{\partial u_j}{\partial x_i}\right)^2 - \tfrac{2}{3}\mu\left(\tfrac{\partial u_k}{\partial x_k}\right)^2\right]\end{aligned} \tag{3}$$

where ρ is the fluid density, u_i are the velocity components, p is the pressure, μ is the dynamic viscosity, T is the temperature, c_p is the specific heat, k is the thermal conductivity and δ_{ij} is the Kronecker delta function. These equations are non-dimensionalized using the channel hydraulic diameter D_h, reference axial velocity u_r, and reference fluid properties ρ_r, μ_r, $c_{p,r}$, k_r. Pressure is normalized by $\rho_r u_r^2$. All other non-dimensional groups are defined based on the reference properties as:

$$\mathrm{Re} = \frac{\rho_r u_r D_h}{\mu_r} \qquad \mathrm{Pr} = \frac{c_{p,r}\mu_r}{k_r} \qquad \mathrm{Ma}^2 = \frac{u_r^2}{\gamma_r R_r T_r} \tag{4}$$

The flow is mass-driven, and the outlet pressure is specified based on the reference density and the wall temperature. Also, zero axial gradients at the outlet are specified for temperature and velocity. At the channel inlet uniform velocity and temperature profiles are specified such that $u = u_{in}$ $v = w = 0$ and $T = T_{in}$. When simulating the variable physical property cases, since the inlet pressure is not known *a priori*, the inlet density is calculated at each iteration and then used to update the inlet velocity for the next iteration. In the case of constant physical properties, the inlet density and velocity will remain constant throughout the solution. The Reynolds number is set constant while comparing the constant and variable-property simulations. Also, when different aspect ratios are studied, in order to make a meaningful comparison between different cases, the hydraulic diameter of the channel is assumed to be constant. This way, the mass flow rate per unit area $\rho u = \mu_r \mathrm{Re}/D_h$ remains the same for different channel aspect ratios.

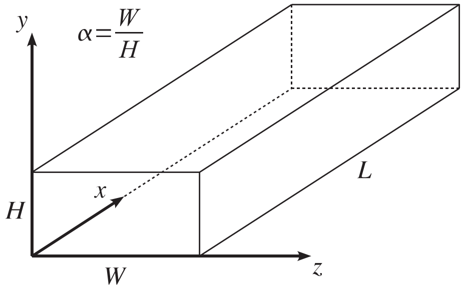

Fig. 1. Channel geometry and the coordinate system.