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A mixed-timescale SGS model for thermal field at various Prandtl numbers

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ABSTRACT

A new subgrid-scale (SGS) model for the thermal field is proposed. The model is an extended version of the mixed-timescale (MTS) SGS model for velocity field by Inagaki et al. (2005), which has been confirmed to be a refined SGS model for velocity field suited to engineering-relevant practical large eddy simulation (LES). In the proposed model for the thermal field, a hybrid timescale between the timescales of the velocity and thermal fields is introduced in a manner similar to velocity-field modeling. Thus, the present model dispenses with an ambiguous SGS turbulent Prandtl number, like the dynamic SGS model. In addition, the wall-limiting behavior of turbulence is satisfied, which is not in the original MTS model, by incorporating the wall-damping function for LES based on the Kolmogorov velocity scale proposed by Inagaki et al. (2010). The model performance is tested in plane channel flows at various Prandtl numbers, and the results show that this model gives the ratio of the timescales between the velocity and thermal fields similar to that obtained using the dynamic Smagorinsky model with locally calculated model parameters. It is also shown that the proposed model predicts better mean and fluctuating temperature profiles in cooperation with the revised MTS model for the velocity field, than the Smagorinsky model and the dynamic Smagorinsky model. The present model is constructed with fixed model parameters, so that it does not suffer from computational instability with the dynamic model. Thus, it is expected to be a refined and versatile SGS model suited for practical LES of the thermal field.

1. Introduction

Several SGS models have been proposed to improve the applicability of large eddy simulation (LES) to complex flows encountered in engineering problems. The best-known is the dynamic Smagorinsky model proposed by Germano et al. (1991), which adjusts the model parameter by itself and dispenses with an explicit wall-damping function that is requisite for the Smagorinsky model. Moin et al. (1991) have generalized the dynamic model for LES of compressible flows and transport of a scalar, so it is available also in thermal-field calculation.

In spite of the remarkable success of the dynamic model, some problems have occurred in its practical use. First, since the model parameter obtained by using the dynamic approach often becomes negative, and/or highly fluctuates in space and time, numerical instability is caused, and the simulation becomes unstable. Although the negative model parameter might represent a transfer of turbulent kinetic energy from the small scales to the resolved scales, i.e., the backscatter phenomenon, Ghosal et al. (1995) demonstrated that the negative model parameter remained negative value for long periods of time unless an *ad hoc* approach of clipping or volume-averaging in calculating the model parameter was used. Even using such an approach for stabilization, numerical instability is often encountered, which restricts the computational time step and brings about an increase in computational cost. The other problem is less predictive accuracy than the Smagorinsky model with the model parameter intentionally optimized for a relevant flow field (Inagaki et al., 2005), at least using the above-mentioned *ad hoc* approach in calculating SGS turbulent viscosity. Moreover, Rasam et al. (2011) have reported the high grid-dependency of the dynamic model. One reason for this is that the damped region of the calculated model parameter near the wall broadens far away from the wall when the grid resolution is not sufficiently high, as reported by Inagaki et al. (2005). To overcome these problems, some studies have been carried out, including that of Meneveau et al. (1996), who proposed a Lagrangian-path averaging method. Although this averaging approach improves the numerical stability, the obtained results heavily depend on the averaging time scale in complex engineering flows.

Ducros et al. (1998) proposed the WALE model that also satisfies the wall-asymptotic behavior. Although some studies reported its usefulness, the model is ordinarily accompanied with a constant SGS Prandtl number, Pr_{SGS}, when modeling the SGS heat flux. Kobayashi (2005) proposed an SGS model based on coherent

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structures, where the second invariant of a velocity gradient tensor normalized by the magnitude of the tensor provides a wall-damping effect, while Yoshizawa's model (Yoshizawa and Horiuti, 1985) is the representative one-equation SGS model, where the transport equation for the subgrid-scale (SGS) kinetic energy, k_{SGS}, is solved. When these SGS models are applied to thermal or scalar field calculation, the use of a constant SGS Prandtl number, Pr_{SGS}, is the conventional approach. However, since the SGS Prandtl number should be varied with the molecular Prandtl number, Pr, and the dissimilarity between the flow and thermal fields, including the distance from the wall (which is apparent in the computational results obtained using the dynamic Smagorinsky model, e.g., Moin et al. (1991)), such a modeling approach for thermal or scalar field, i.e., use of a constant Pr_{SGS}, is considered questionable in light of the accurate prediction of SGS heat flux.

On the other hand, Inagaki et al. (2005) proposed a new SGS model constructed with the concept of mixed timescale, which makes it possible to use fixed model parameters like the WALE and the coherent structure models. In general, the use of fixed model parameters assures computational stability. The model performance was widely tested in several basic flows and engineering-relevant flows (Inagaki et al., 2003, 2005, 2010), and it was confirmed that both the prediction accuracy and the computational stability are higher than with the dynamic Smagorinsky model. The concept of mixed timescale is considered applicable also to thermal-field modeling, which implies that the resultant SGS Prandtl number could vary to express the dissimilarity between velocity and thermal fields like the dynamic model. Thus, if an extended version of the mixed-timescale (MTS) model were successfully constructed, it could be a refined SGS model suited for practical LES of the thermal or scalar field.

Meanwhile, the MTS model does not follow the correct wall-limiting behavior; $\nu_t \sim y^2$ in the model. This shortcoming probably becomes discernible in the calculation of a thermal field, though it does not lead to the reduction of prediction accuracy in calculating flow fields. However, this problem could be settled by using the MTS model in combination with the explicit wall-damping function for LES proposed by Inagaki (2011). This wall-damping function uses the Kolmogorov velocity scale instead of wall-friction velocity in the calculation of dimensionless wall-distance so as to eliminate the well-known problem of van Driest's wall-damping function. This type of wall-damping function was first proposed by Abe et al. (1994) in their k–ε model, and its effectiveness in the Reynolds averaged Navier–Stokes (RANS) model has been ascertained in many subsequent studies including one of the latest RANS models by Abe et al. (2003). In LES, the dissipation rate of turbulence energy, ε, which is needed to calculate the Kolmogorov velocity scale, is generally not solved, unlike in the RANS simulations. Therefore, the use of the Kolmogorov velocity scale in LES is not straightforward. To resolve this issue, Inagaki (2011) proposed a conversion method for estimating the Kolmogorov velocity scale in LES, and the estimated one is utilized in the wall-damping function. Its validity was assessed in canonical channel flows, and also a backward-facing step flow, and the results showed that the damping effect of the developed wall function is almost independent of the grid resolution and the Reynolds number, and is appropriate even in the flow with flow separation and reattachment.

Thus, our objective is to propose a new SGS model for the thermal field based on the concept of mixed-timescale, which provides high prediction accuracy and high computational stability in cooperation with the MTS model for the velocity field, and is capable of expressing the dissimilarity between velocity and thermal fields. In the model, the hybrid timescale between the timescales of the velocity and thermal fields is introduced in order to eliminate an ambiguous SGS turbulent Prandtl number. The model performance is tested in plane channel flows at different Prandtl numbers and compared to the results with the dynamic model and standard Smagorinsky model.

2. Governing equations and mixed-timescale SGS models

2.1. Governing equations

The basic equations are the filtered Navier–Stokes, continuity, and energy (temperature transport) equations for an incompressible fluid given as follows:

$$\frac{\partial \bar{u}_j}{\partial x_j} = 0, \tag{1}$$

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_j \bar{u}_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j}, \tag{2}$$

$$\frac{\partial \bar{\theta}}{\partial t} + \frac{\partial \bar{u}_j \bar{\theta}}{\partial x_j} = \frac{\partial q_j}{\partial x_j} + \alpha \frac{\partial^2 \bar{\theta}}{\partial x_j \partial x_j} + Q_{in}, \tag{3}$$

$$\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j, \tag{4}$$

$$q_j = \overline{\theta u_j} - \bar{\theta}\bar{u}_j, \tag{5}$$

where $(^{-})$ denotes the grid-filtering operator, and Q_{in} represents the inner heat source. τ_{ij} and q_j are the SGS stress and the SGS heat flux, respectively, which should be modeled. All the SGS models tested in this paper are based on the eddy viscosity concept:

$$\tau^*_{ij} = -2\nu_t \bar{S}_{ij}, \quad \bar{S}_{ij} = \frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right), \tag{6}$$

$$q_j = -\alpha_t \bar{G}_j, \quad \bar{G}_j = \frac{\partial \bar{\theta}}{\partial x_j}, \tag{7}$$

where $\tau^*_{ij} = \tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk}$.

2.2. Revised MTS model for velocity field

The wall-asymptotic behavior of the original MTS model (Inagaki et al., 2005) is $\nu_t \sim y^2$, which does not follow the correct behavior, $\nu_t \sim y^3$. Although this property does not lead to the reduction of prediction accuracy in calculating flow fields, this shortcoming probably becomes discernible in calculating thermal fields, especially at high Prandtl number conditions where the turbulent heat flux rapidly decreases near the wall because of the thinner thermally conductive sublayer than the viscous sublayer. Thus, the MTS model is modified to satisfy correctly the wall-limiting behavior of turbulence as follows:

$$\nu_t = C_{MTS}\, f_\mu\, k_{es}\, \tau_u, \tag{8}$$

$$k_{es} = C_{kes}(\bar{u}_k - \hat{\bar{u}}_k)^2, \tag{9}$$

$$\tau_u^{-1} = \frac{1}{2}\{\tau_\Delta^{-1} + \tau_S^{-1}\}, \tag{10}$$

$$\tau_\Delta = \frac{\bar{\Delta}}{\sqrt{k_{es}}}, \quad \tau_S = \frac{C_{Tu}}{|\bar{S}|}, \tag{11}$$

$$|\bar{S}| = \sqrt{2\left(\bar{S}_{ij}\bar{S}_{ij}\right)}, \tag{12}$$

$$C_{MTS} = 0.025,\ C_{Tu} = 10,\ C_{kes} = 1 \tag{13}$$

$$f_\mu = \left[1 - \exp\left\{-\left(\frac{y'_\varepsilon}{A_\mu}\right)^{\frac{4}{3}}\right\}\right]^{\frac{1}{2}}, \tag{14}$$

$$y'_\varepsilon = u'_\varepsilon y_n/\nu,\ u'_\varepsilon = (\nu\varepsilon_{SGS})^{\frac{1}{4}}\left(\frac{C_l y}{\bar{\Delta}}\right)^{\frac{1}{2}}, \tag{15}$$

$$\varepsilon_{SGS} = C_\varepsilon \frac{k_{es}^{3/2}}{\bar{\Delta}} + 2\nu \frac{\partial\sqrt{k_{es}}}{\partial x_j}\frac{\partial\sqrt{k_{es}}}{\partial x_j}, \tag{16}$$

$$A_\mu = 2,\ C_l = 4,\ C_\varepsilon = 0.835. \tag{17}$$