International Journal of Heat and Fluid Flow 29 (2008) 1462–1468

Contents lists available at ScienceDirect

International Journal of Heat and Fluid Flow

journal homepage: www.elsevier.com/locate/ijhff

Numerical investigation of the flow of a glass melt through a long circular pipe

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ARTICLE INFO

Article history:
Received 13 March 2007
Received in revised form 23 May 2008
Accepted 9 June 2008
Available online 9 August 2008

Keywords:
Laminar flow
Pipe flow
Numerical simulation

ABSTRACT

This work is concerned with the comparison between a two-dimensional axisymmetric simulation of glass melt flowing through a pipe with a circular cross-section and a one-dimensional model studied by Gießler et al. [Gießler, C., Lange, U., Thess, A., 2007. Nonlinear laminar pipe flow of fluids with strongly temperature-dependent material properties. Phys. Fluids 19, 043602]. The fluid is supposed to be heated by internal electromagnetic (Joule) heating and cooled at the wall by convection. The exponential temperature dependence of the viscosity and the electrical conductivity are fully taken into account. The two models are being compared over a range of values of two different parameters. We find a very good agreement for moderate and high thermal conductivities or in the case of dominating heating. For strong cooling and low thermal conductivities, differences between the one-dimensional model and the two-dimensional axisymmetric simulation occur. The comparison shows that in the latter case the formation of strong radial temperature variations and the resulting radial variation of the viscosity lead to a divergence between the two-dimensional simulation and the one-dimensional model.

1. Introduction

During the production of glass usually Forehearth and Feeder systems are used to transport and condition the glass melt (Noelle, 1997). These are typically pipes or ducts connecting the melting furnace and forming device where the melt is cooled down from the refining temperature to a suitable forming temperature. This is often realized by applying an electrical current either directly into the melt or if the walls are conductive, upon the walls of the pipe. The temperature variation within the melt leads to a large change of various material properties which can lead to unintended inhomogeneities of the final glass product.

In order to control the cooling process we have to obtain a deeper understanding of the behaviour of the system. An accurate prediction of the pipe flow is necessary including all relevant temperature-dependent material parameters. The goal of our work is to show numerically the complex flow structure of a two-dimensional axisymmetric pipe flow if the temperature-dependent material properties of glass melts are included.

There have been several works mainly in geological science focusing on temperature-dependent viscosity. In Richardson (1986), Whitehead and Helfrich (1991), Helfrich (1995), and Wylie and Lister (1995), one-dimensional models of lava flow in slots, ducts or pipes with cooled walls had been derived. These studies observed that the temperature-dependent viscosity can lead to a dramatic modification of the laminar flow characteristics. Bifurcation develops for sufficiently large viscosity differences – for a given pressure drop three steady state solutions were found. Similar results were found for a pipe flow of glass melts in Lange and Loch (2002) and reported from glass production (Lange, private communication). In all these works only non-heated flows in cooled tubes were considered. Heating, however, has not been considered at all. A recent theoretical contribution (Gießler et al., 2007) examined the influence of coupled wall heat loss and internal volumetric heating. Furthermore, this work included the full non-linear temperature dependence of the viscosity and electrical conductivity. In this one-dimensional model of glass melt the flow is driven by a constant external force density which is acting along the whole pipe. Additionally, the flow is influenced by temperature variations due to wall heat loss, internal heating, advection, and diffusion. The model allows the calculation of the mean velocity and the mean temperature for a given set of parameters. While a bifurcation develops when the fluid is cooled, a non-linear laminar flow characteristic develops when the fluid is heated. However, this analytical model neglects the dependence of velocity and temperature on the radial coordinate which may influence the flow as well.

The goal of the present work is to validate this one-dimensional model and to analyse the influence of the dependence of velocity and temperature on the radial coordinate. In order to achieve this goal we systematically perform numerical parameter studies of the pipe flow model using a two-dimensional axial configuration. We would like to find the range of validity for the one-dimensional model. Furthermore, we will explain physically deviations of the numerically and analytically obtained results.

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0142-727X/$ - see front matter © 2008 Elsevier Inc. All rights reserved.
doi:10.1016/j.ijheatfluidflow.2008.06.001

Nomenclature

A	glass-specific viscosity parameter	T_{in}	inlet temperature (K)
B	glass-specific viscosity parameter (K)	T_{out}	mean outlet temperature (K)
C	glass-specific viscosity parameter (°C)	$\boldsymbol{u}(x,r)$	velocity field (m/s)
c_p	heat capacity (J/(kg K))	u_m	mean velocity (m/s)
E	glass-specific constant	x,r	system of coordinates (m)
F	glass-specific constant (K)	λ	thermal conductivity (W/m K)
G	glass-specific constant (°C)	η	dynamic viscosity (Pa s)
$\boldsymbol{J}$	current density (A/m^2)	η_0	glass-specific viscosity parameter (Pa s)
L	length of the pipe (m)	α	heat-transfer coefficient (W/m^2 K)
p_{diff}	mean pressure difference (Pa)	ρ	density (kg/m^3)
R	radius of the pipe (m)	σ	electrical conductivity (S/m)
T	temperature (K)	σ_0	glass-specific constant (S/m)
T_∞	ambient temperature (K)		

2. Mathematical formulation and numerical method

2.1. Definition of the model

We consider a laminar and steady flow of a viscous electrically conducting fluid in a pipe with circular cross-section driven by a pressure difference between the inlet and the outlet of the pipe. The pipe has the length L and the radius $R \ll L$ as shown in Fig. 2-1. At the inlet of the pipe the temperature T_{in} and the parabolic flow profile with the mean velocity $u_m = u_{max}/2$ are given. An external homogenous horizontal electric current density $\boldsymbol{J}$ acts upon the entire circular pipe. The Joule heat flux density $J^2/\sigma(T)$ acts as an energy source.

The highly viscous fluid with constant density ρ is supposed to have strongly temperature-dependent viscosity $\eta(T)$ and electrical conductivity $\sigma(T)$. During the simulations the temperature-dependent viscosity equation,

$$\eta(T) = \eta_0 \cdot \exp(-A + B/(T + C)), \tag{1}$$

and the equation for the temperature-dependent electrical conductivity

$$\sigma(T) = \sigma_0 \cdot \exp(E - F/(T + G)) \tag{2}$$

are used. The constant parameters A, B, C, E, F, G are specific to the considered glass melt. While the viscosity decreases as the temperature increases, the electrical conductivity increases significantly with the temperature. As C is typically negative, the viscosity law Eq. (1) only makes sense for $T > -C$ as $\eta \to \infty$ for $T \to |C|$.

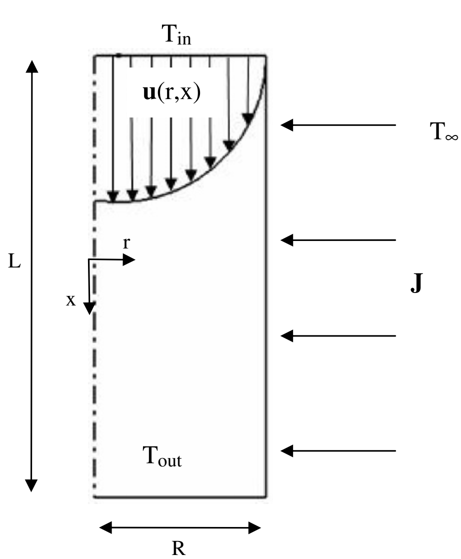

Fig. 2-1. Sketch of the considered flow in a circular pipe.

The steady pipe flow is governed by the steady Navier–Stokes equation,

$$\rho u \cdot (\nabla u) = -\nabla p + \nabla \cdot [\eta(\nabla u + (\nabla u)^T)], \tag{3}$$

and the condition of incompressibility

$$\nabla \cdot u = 0. \tag{4}$$

The left hand-side of the Stokes Eq. (3) represents the advection of the velocity field and the right hand-side represents the driving pressure gradient and the internal friction. We like to emphasize, that the velocity field $\boldsymbol{u} = u_r(x,r)\boldsymbol{e}_r + u_x(x,r)\boldsymbol{e}_x$ is not only a function of the radial coordinate r. As the viscosity is a function of the temperature and therefore depends both on r and x, the velocity $\boldsymbol{u}$ depends on the axial position as well, e.g. $\boldsymbol{u} = \boldsymbol{u}(x,r)$. As a result the flow is not fully developed. The applied boundary conditions of the two-dimensional axial model are

$$\frac{\partial u_x}{\partial r} = 0, \quad u_r = 0, \quad \text{for } r = 0, \tag{5a}$$

$$u_r = u_x = 0 \quad \text{for } r = R, \tag{5b}$$

$$u_x = 2 \cdot u_m \cdot \left(1 - \left(\frac{r}{R}\right)^2\right), \quad u_r = 0 \quad \text{for } x = 0, \tag{5c}$$

$$u_r = 0, \quad p = 0 \quad \text{for } x = L. \tag{5d}$$

Let us note that the boundary condition at the outlet, Eq. (5d), is the outflow condition which is provided by the commercial tool Comsol.

To calculate the temperature distribution in the pipe the steady energy equation

$$\rho c_p u \cdot \nabla T = \frac{J^2}{\sigma(T)} + \lambda \cdot \nabla^2 T \tag{6}$$

applies with the boundary conditions given by

$$\frac{\partial T}{\partial r}(0,x) = 0 \quad \text{for } r = 0, \tag{7a}$$

$$-\lambda \frac{\partial T(R,x)}{\partial r} = \alpha(T(R,x) - T_\infty) \quad \text{for } r = R, \tag{7b}$$

$$T(r,0) = T_{in} \quad \text{for } x = 0, \tag{7c}$$

$$\boldsymbol{e}_x \cdot (-k\nabla T) = 0 \quad \text{for } x = L. \tag{7d}$$

The heat convection is specified on the left-hand side of the energy Eq. (6) with the constant density ρ and the constant heat capacity c_p. The first term on the right-hand side represents the generation of heat by the Joule effect according to $J^2/\sigma(T)$. The second term specifies the heat conduction with constant thermal conductivity