International Journal of Heat and Fluid Flow 31 (2010) 217–226

Contents lists available at ScienceDirect

International Journal of Heat and Fluid Flow

journal homepage: www.elsevier.com/locate/ijhff

# Numerical investigation of double-diffusive (natural) convection in vertical annuluses with opposing temperature and concentration gradients

Sheng Chen [a,b], Jonas Tölke [b], Manfred Krafczyk [b,*]

[a] State Key Lab of Coal Combustion, Huazhong University of Science and Technology, Wuhan 430074, China
[b] Institute for Computational Modeling in Civil Engineering, Technical University, Braunschweig 38106, Germany

ARTICLE INFO

*Article history:*
Received 8 April 2009
Received in revised form 21 December 2009
Accepted 26 December 2009
Available online 8 February 2010

*Keywords:*
Lattice Boltzmann model
Double-diffusive convection

ABSTRACT

Double-diffusive convection in vertical annuluses with opposing temperature and concentration gradients is of fundamental interest and practical importance. However, available literature especially for higher Rayleigh numbers beyond $Ra \leqslant 10^5$ is sparse. In this study, we investigated double diffusion induced convection up to $Ra = 10^7$ using a simple lattice Boltzmann model. Thanks to the good stability of the present model, a modest grid resolution is sufficient for the present simulations. The influences of the ratio of buoyancy forces $0.8 \leqslant N \leqslant 1.3$, the aspect ratio $0.5 \leqslant A \leqslant 2$ and the radius ratio $1.5 \leqslant K \leqslant 3$ on heat and mass transfer characteristics are discussed in detail.



## 1. Introduction

Double-diffusive convection, i.e. flows generated by buoyancy due to simultaneous temperature and concentration gradients are ubiquitous in natural as well as technical systems. In nature such flows are frequently encountered in oceans, lakes, solar pounds, shallow coastal waters and the atmosphere. In industry examples include chemical processes, crystal growth, energy storage, material processing such as solidification, food processing etc. For a review of the fundamental work in this area, see Turner (1974) and Schmitt (1994).

Available studies related to double-diffusive convection are mostly concerned with rectangular cavities. Pioneering experiments were carried out by Kamotani et al. (1985). Their work showed that when a stable stratified solution is heated from one side, multicellular flow structures can be observed. Later, Gobin and Bennacer (1996) identified the different regimes dominated by thermal or solutal effects in terms of the buoyancy ratio and the Lewis number based on numerical simulations. The instability of double-diffusive convection has been studied analytically by Bardan et al. (2000). More recently, Sezai and Mohamad (2000) and Bergeon and Knobloch (2002) carried out three-dimensional numerical simulations on double-diffusive convection in cubic cavities, to cite only a few. However, there are only few studies on double-diffusive convection in vertical annuluses (Retiel et al., 2006), although convection related phenomena in vertical annuluses are scientifically more interesting than those in rectangular/cubic cavities (Turner, 1974). Shipp et al. (1993), Shipp et al. (1993) investigated thermosolutal convection in concentric annular cavities at low and moderate Lewis numbers. Bennacer et al. (2000) simulated the thermosolutal convection in vertical annular cavities containing a porous medium. Recently Retiel et al. (2006) investigated the effect of the curvature ratio on convectional patterns. The latest work on double-diffusive convection in vertical annuluses was conducted by Bennacer et al. (2009). In their work, they studied the Soret effect for double-diffusive convection in detail. In almost all previous works on this field, the Rayleigh number was relatively low ($Ra \leqslant 10^5$).

In the present work, the double-diffusive convection in vertical annuluses with opposing temperature and concentration gradients is reported for higher Rayleigh numbers up to $Ra = 10^7$ . The influences of the ratio of buoyancy forces $0.8 \leqslant N \leqslant 1.3$, the aspect ratio $0.5 \leqslant A \leqslant 2$ and the radius ratio $1.5 \leqslant K \leqslant 3$ on heat and mass transfer characteristics are discussed in detail in this study. In order to numerically solve the governing equations for such double-diffusive convection, a simple lattice Boltzmann (LB) model, which is an extension of the model proposed in our previous works (Chen et al., 2008; Chen et al., 2009), is employed in this paper. The present model possesses three obvious advantages inherited from our previous models (Chen et al., 2008; Chen and Tolke, 2009):

1. The present model is algorithmically simple, which is an attractive advantage for both practitioners and novices.
2. Tts stability and low numerical viscosity allows the use of relatively coarse grids for flow with high Rayleigh numbers which reduces computational costs. and

* Corresponding author.
*E-mail addresses:* shengchen.hust@gmail.com (S. Chen), kraft@irmb.tu-bs.de (M. Krafczyk).

0142-727X/$ - see front matter © 2010 Elsevier Inc. All rights reserved.
doi:10.1016/j.ijheatfluidflow.2009.12.013

**Nomenclature**

| | |
|---|---|
| $c$ | fluid particle speed |
| $\Omega_k$ | collision term in Eq. (31) |
| $D$ | coefficient related to Eq. (31) |
| $\vec{u}$ | fluid velocity vector |
| $\vec{e}_k$ | discrete velocity |
| $\Upsilon_{o,k}, \Omega'_k$ | source terms in Eqs. (25), (31) |
| $\vec{g}$ | gravity |
| $g_k,\ f_k$ | distribution function for Eqs. (17), (18) and Eq. (19) |
| $g_k^{eq},\ f_k^{eq}$ | equilibrium distribution function for Eqs. (1)–(3) |
| $H$ | height of simulation domain |
| $S$ | Svanberg vorticity |
| $T$ | temperature |
| $R$ | radius |
| $N$ | ratio of buoyancy forces |
| $Pr$ | Prandtl number |
| $Le$ | Lewis number |
| $Ra$ | Rayleigh number |
| $p$ | pressure |
| $K$ | curvature ratio |
| $A$ | aspect ratio |
| $\vec{x}$ | phase space |

*Greek symbols*

| | |
|---|---|
| $\Delta x,\ \Delta t$ | grid spacing, time step |
| $\kappa$ | thermal conductivity |
| $\alpha$ | expansion coefficient |
| $\nu$ | kinematic viscosity |
| $\omega,\ \psi$ | vorticity, streamfunction |
| $\tau$ | relaxation time for Eq. (25) |
| $\tau_\psi$ | relaxation time for Eq. (31) |
| $\rho$ | density |
| $\zeta$ | dimensionless time |
| $\zeta_k,\ \xi_k$ | weights for equilibrium distribution function |
| $\delta,\ \chi$ | coefficients in Eqs. (29) and (30) |
| $\mu$ | dynamic viscosity |

*Subscripts and superscripts*

| | |
|---|---|
| $\mu$ | dynamic viscosity |
| $o$, $i$ | outer, inner |
| 0 | reference value |
| $k$ | discrete velocity direction |
| $c$ | concentration |
| $T$ | temperature |

3. The derivation of the present model is quite straightforward as is often observed for kinetic models (see also the recent review Yu et al., 2003).

## 2. Governing equations for double-diffusive convection in vertical annuluses

The configuration of the vertical annulus is illustrated in Fig. 1. The inner wall with the radius $R_i$ and the outer wall with $R_o$. $K = R_o/R_i$ is the radius ratio. The aspect ratio is defined as $A = H/(R_o - R_i)$, where $H$ is the height of the annular cavity.

Based on the Boussinesq assumption, the primitive-variables-based governing equations for double-diffusive convection in the cylindrical coordinate system can be written as (Retiel et al., 2006; Shipp et al., 1993; Shipp et al., 1993; Chamkha and Al-Naser, 2002)

$$\frac{1}{r}\frac{\partial(ru)}{\partial r} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\nabla^2 u, \tag{2}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\nabla^2 w + g\alpha_T(T - T_0) - g\alpha_c(C - C_0), \tag{3}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \kappa\nabla^2 T, \tag{4}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial r} + w\frac{\partial C}{\partial z} = D\nabla^2 C, \tag{5}$$

where

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial z^2}.$$

$u$ and it w are radial and axial velocity components, $p$ is the pressure, $T$ is the temperature, $C$ is the concentration, $\nu$ is the kinematic viscosity, $g$ is the gravitational acceleration along the negative $z$-axis, $\kappa$ is the thermal conductivity, $\rho$ is the density, $D$ is the species diffusivity, $\alpha_T$ and $\alpha_c$ represent the coefficients of thermal expansion and compositional expansion respectively.

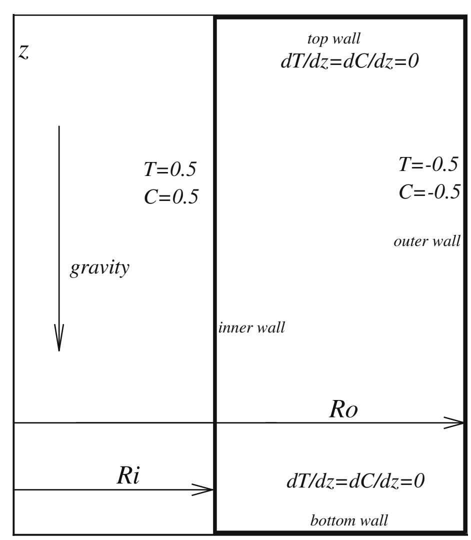


**Fig. 1.** Configuration of the computational domain and boundary conditions.

For double-diffusive convection in a cylindrical coordinate system, computation time can be reduced if the problem is reformulated so that the three variables $u$, $w$, $p$ are eliminated in favor of the vorticity $\omega$ and the Stokes streamfunction $\psi$ (Chen et al., 2008; Chamkha and Al-Naser, 2002; Langlois, 1985; Chen et al., 2008), which are defined as

$$\omega = \frac{\partial w}{\partial r} - \frac{\partial u}{\partial z}, \tag{6}$$

$$u = \frac{1}{r}\frac{\partial \psi}{\partial z}, \tag{7}$$

$$w = -\frac{1}{r}\frac{\partial \psi}{\partial r}. \tag{8}$$