International Journal of Heat and Fluid Flow 30 (2009) 54–65

Contents lists available at ScienceDirect

International Journal of Heat and Fluid Flow

journal homepage: www.elsevier.com/locate/ijhff

A new extension for k–ω turbulence models to account for wall roughness

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ARTICLE INFO

Article history:
Received 23 July 2007
Received in revised form 16 July 2008
Accepted 23 September 2008
Available online 21 November 2008

Keywords:
Boundary layer
Turbulence modelling
Wall roughness
Equivalent sand grain
Skin friction

ABSTRACT

This paper presents a new extension for k–ω turbulence models to account for surface roughness for transitionally and fully rough surfaces. It is based on the equivalent sand grain approach and accounts for theoretical considerations on the log-layer solution for fully rough surfaces. An appropriate behaviour for transitional roughness is achieved by means of wall values for k and ω which depend on the roughness Reynolds number. In the limit of vanishing roughness, the smooth wall boundary condition is recovered. For the full range of roughness Reynolds numbers the new roughness modification gives very successful predictions for a variety of flat plate turbulent boundary layer flows and for the pipe flow experiments by Nikuradse. The new method allows for the simulation of flows over rough surfaces at the same grid resolution requirements as for smooth walls. Thereby the extremely fine near-wall mesh resolution required by the Wilcox roughness modification is avoided. Secondly the new roughness modification gives significantly improved predictions in skin friction for transitional roughness Reynolds numbers compared to the roughness extension by Wilcox. Thirdly, the new roughness extension does not require a modification of the SST k–ω model, whereas a modification is necessary if the roughness extension by Wilcox is used. Finally the new method is applied successfully to predict the aerodynamic effects of surface roughness on the flow past an airfoil in highlift conditions.

1. Introduction

Most wall-bounded flows of engineering interest are turbulent in character. In many situations at least parts of the surface are rough, e.g., aerodynamic flows over airfoils with icing or turbine blades with surface roughness due to manufacturing imperfections or as a longterm result of erosion by impinging combustor air.

The accurate and reliable prediction of the effects of surface roughness on fluid flow and heat transfer are of great interest for engineers using CFD as a major design tool. A survey on different approaches for turbulence model modifications to account for surface roughness can be found e.g., in Patel (1998), and Aupoix and Spalart (2003). In the present paper, the "equivalent sand grain approach" is considered, which is due to the work of Nikuradse (1933). This approach uses a theoretical roughness length called equivalent sand grain roughness. Nikuradse performed experiments with pipes with sand glued to the wall as densely as possible, and for the equivalent sand grain size he used the size of the sieve. If surface roughness comes from regular arrays of discrete three-dimensional roughness elements of a certain geometry such as cones, hemispheres, etc., or from a stochastic roughness distribution, then the corresponding equivalent sand grain roughness height has to be computed from the real, geometrical roughness size using an empirical correlation, see e.g. Schlichting (1968), Dirling (1973).

The experimental data by Nikuradse are still of immeasurable value for the design and validation of roughness modifications for turbulence models. In pipe flow, friction is related to the drop in pressure over an axial distance, which can be measured very accurately. Nikuradse proposed empirical relations for the skin friction (to be more precise: for the friction factor) and for the shift of the velocity profile in the logarithmic layer as a function of the equivalent sand grain roughness. Moreover, experimental results from several research groups for flat plate turbulent boundary layer flow with surface roughness provide additional data for skin friction and partially also for the shift of the log-layer profiles for velocity.

For boundary layers over flat plates with surface roughness, local skin friction coefficients are determined from the Reynolds shear stresses and mean velocities, measured at a distance above the crests of the roughness elements where $-\overline{u'v'}/u_\infty^2$ is 96–98% of $c_f/2$. Using hot-wire anemometry, the uncertainty in $-\overline{u'v'}$ and thus in c_f is about $\pm 10\%$, see Ligrani and Moffat (1986) and references therein.

In computational models which use the equivalent sand grain approach, the rough surface is replaced by an effective, smooth surface, on which modified boundary conditions are imposed. For

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doi:10.1016/j.ijheatfluidflow.2008.09.009

Nomenclature

B	shift in the log-law for rough walls
C	shift in the log-law for smooth walls, $C = 5.1$
c	airfoil chord length
c_f	skin friction coefficient $c_f = \tau_w/((1/2)\rho u_\infty^2)$
d_0	offset in the wall distance to account for wall roughness
L	length of the flat plate (in [m])
k	turbulent kinetic energy in the k–ω model
k_r	equivalent sand grain roughness height
R	radius of the pipe in Nikuradse experiments
u	streamwise velocity component
u_τ	friction velocity
x	streamwise position for flow over flat plate (in [m])
y	distance to the nearest wall

Greeks

α	angle of incidence for airfoil flows
β_k	constant in the k–ω model, $\beta_k = 0.09$
κ	von Kármán constant
ν	kinematic viscosity
ν_t	eddy or turbulent viscosity
ρ	density
τ_w	wall shear stress
ω	turbulence frequency in the k–ω model

Subscripts, superscripts

$+$	variable in wall scaling
CL	centerline velocity in pipe flow
∞	freestream value
w	wall value

Abbreviations

RANS	Reynolds averaged Navier–Stokes equations
SA model	Spalart–Allmaras turbulence model

low-Re turbulence models see Aupoix and Spalart (2003), Durbin et al. (2001) and for wall-functions see Suga et al. (2006) and references therein. The design of roughness modifications has to ensure both (i) predictions of c_f close to experimental data and (ii) prediction of profiles for velocity and turbulence quantities in agreement with experimental results and empirical theory. For fully rough surfaces, the design of roughness modifications can be guided by theoretical considerations regarding the log-layer solution, see e.g. Kays and Crawford (1993) pp. 230. On the other hand, regarding the near-wall behaviour of mean velocity and turbulence quantities, the available experimental data are very limited and at present no successful empirical relations exist.

Recently, two extensions of the Spalart–Allmaras turbulence model (abbreviated SA model) to account for wall roughness have been proposed, developed independently by Boeing and ONERA, see Aupoix and Spalart (2003). The underlying ideas used for the SA model are valuable also for the design of roughness modifications for other low-Re RANS turbulence models. The two roughness modifications yield similar predictions, and are in fair agreement with experimental data. In this paper, only the Boeing approach is considered, as it is very suitable for parallel, unstructured CFD methods.

A rough wall modification for the two layer k-ϵ model has been proposed in Durbin et al. (2001). The approach uses a calibration procedure for transitional roughness values. Thereby the model is designed to predict the shift of the log-layer profiles for velocity in agreement with the empirical relation by Ligrani and Moffat (1986).

An alternative approach was pursued by Suga et al. (2006), who proposed an analytical wall-function for turbulent flow and heat transfer over smooth and rough walls. Good results for equilibrium boundary layer flows and for flows with separation and reattachment over a sand dune and a sand-roughened ramp are shown. Interestingly, this method allows also to take into account how roughness disrupts the viscous sublayer. This might become even more interesting, if more detailed experimental data for the near-wall region, i.e., $y^+ < 50$, of rough surfaces are available, in particular including effects of a non-small pressure gradient.

The present paper is dedicated to roughness extensions for k–ω type turbulence models which are so-called low-Re models, i.e., the momentum and turbulence model equations are integrated down to the wall. Despite recent advances in the design of wall-functions for aerodynamic flows, see e.g., Medic et al. (2005), Knopp et al. (2006), aerodynamic and turbo-machinery industry still mostly rely on low-Re models at least during the final design stage due to their very high accuracy demands. In particular for aeronautical flows, turbulence models of k–ω type and primarily the SST model by Menter (1993) and also the Spalart–Allmaras model are very popular, see e.g. Vassberg et al. (2007).

First, the well-known roughness modification by Wilcox (1998) is reviewed. Surprisingly, the validation of this model extension is very limited in literature, even in Wilcox (1998) and the cited references. Moreover, it is interesting to note that the formulation in Wilcox (1998) has been modified slightly in Wilcox (2006). In Patel and Yoon (1995), Patel (1998), some results for velocity profiles and friction factor for pipe flow and few results for fully developed channel flow are shown. Hellsten (1997) uses the roughness extension by Wilcox for two variants of the original Wilcox k–ω model, viz., the Menter BSL and the SST k–ω model, and he shows some results for boundary layers over flat plates.

The present investigation addresses two major shortcomings of this approach. The first disadvantage is the very fine near-wall mesh resolution required. Both authors Patel and Yoon (1995) and Hellsten (1997) state that for surface roughness much finer near-wall grids are required than for smooth walls. This constraint increases the computational costs for the appropriate usage of this roughness model in terms of numerical error significantly. Additionally, this can lead to severe problems for the mesh generation in complex geometries, in particular for high Reynolds number flows.

The second shortcoming is that for transitionally rough surfaces the predictions for skin friction are not fully satisfying. Both authors Patel (1998) and Hellsten (1997) consider only the case of a constant Reynolds number (based on the free-stream velocity and the length of the plate) and vary the equivalent sand grain roughness size. Instead, in the present paper the case by Ligrani and Moffat (1986) is considered, in which the roughness height is held constant and the roughness Reynolds number is changed by varying the onflow velocity. This test case is very sensitive for prediction of skin friction.

The third shortcoming is that the Wilcox roughness modification cannot be used in conjunction with the original SST model and requires an additional modification of the SST model in order to prevent the limitation of the eddy viscosity and hence of the modeled shear-stress from being activated in the near-wall region, as shown in Hellsten (1997). This may be seen as an additional indication that the roughness extension by Wilcox is not completely sound.