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# A realisable non-linear eddy viscosity/diffusivity model for confined swirling flows

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ABSTRACT

A non-linear eddy viscosity/diffusivity model for turbulent flows is presented, featuring quadratic constitutive relationships for both Reynolds stresses and scalar fluxes. Model coefficients are defined by enforcing compliance with fundamental experimental evidence, and realizability of both the velocity and scalar fields, which is achieved by making coefficients depend upon an appropriately defined strain parameter. The model is also shown to satisfy joint-realizability. The model is extensively tested against experimental results for confined swirling flows, encompassing a wide range of values of the swirl number, momentum and density ratios. The results unambiguously indicate a remarkable, uniform improvement over standard modelling. Further, previous work on the subject of nonlinear models is reviewed.



## 1. Introduction

Non-linear eddy viscosity models (NLEVMs) appear as potential candidates to replace the well-tried $k$–$\epsilon$ model (Jones, 1971; Jones and Launder, 1972) (with minor re-optimisation of the model constants as in Launder and Spalding (1974)) as a 'workhorse' for the computation of turbulent flows. The $k$–$\epsilon$ and other linear eddy viscosity models are known to exhibit fundamental shortcomings, particularly in their inability to reproduce flows featuring recirculation and/or swirl, streamline curvature and secondary flows in non-circular ducts. Similar deficiencies are evident in flows involving scalar transport related in particular to the largely underestimated ratio of stream wise to transverse turbulent fluxes in heated/cooled channel or pipe flows and scalar fluctuations in buoyant flows that are also poorly reproduced. To correct this behaviour, non-linear models, here termed non-linear eddy diffusivity models (NLEDMs), have been proposed. However, finding a suitable replacement for the standard $k$–$\epsilon$ model appears to be a major challenge; despite the above mentioned weaknesses, it nonetheless features undeniable virtues. These are related to its relative ease of use and robustness and to the fact that it is well-calibrated, so that it leads to acceptable results in many cases. This is in spite of the fact that while a term-by-term analysis of the model undoubtedly reveals inadequacies, the resulting negative impact on the quality of predictions is limited, due to compensating errors. A plethora of NLEVMs and (to a smaller extent) NLEDMs have been proposed in recent years. Such a proliferation is clearly a consequence of the large number of undetermined coefficients appearing in the non-linear expansions of Reynolds stresses and scalar fluxes, which can be specified according to different criteria. A categorization of these models can be attempted, based on the following criteria:

(a) Possible inclusion of higher-order derivatives.
(b) Choice of variables to identify turbulent velocity and length scales.
(c) Order of the polynomial expansion.
(d) Relationship to second-moment models.

As far as item (a) is concerned, it can be observed that by far the vast majority of non-linear models adopt forms which only feature the first derivatives of the mean velocity components and of the mean scalar. However, a small number of NLEVMs (Speziale, 1987; Huang and Rajagopal, 1996) start from a form including the second-derivatives of the mean velocity components. The latter choice, although supposedly involving some advantage raises the order of the resulting RANS equations above that of the original Navier–Stokes equation with the consequence that boundary conditions are required for the mean velocity components and their spatial gradients.

Item (b) also features two options, with one being overwhelmingly more popular than the other. In fact, whereas practically all models choose the square root of the turbulent kinetic energy $\sqrt{k}$ as a turbulent velocity scale, either the mechanical dissipation rate $\epsilon$ or a pseudo-vorticity $\omega$ can be used to construct a turbulent time scale, with the latter approach representing an extension of the (linear) $k$–$\omega$ model (Saffman, 1970; Wilcox, 1993). The vast

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majority of non-linear models adopt $\epsilon$ as the second variable, whilst a few prefer $\omega$ (Wilcox, 1993; Abdon and Sundén, 2001; Merci et al., 2001; Song et al., 2001). One-equation NLEVMs, involving only an equation for $k$, have also been proposed (Spalart and Shur, 1997; Spalart, 2000).

The near majority view above is not replicated for the two remaining items. In particular, as far as item (c) is concerned, NLEVMs have been proposed adopting quadratic (Speziale, 1987; Gatski and Speziale, 1993; Shih et al., 1993, 1995; Spalart and Shur, 1997; Luo and Lakshminarayana, 1997; Gatski and Jongen, 2000; Rumsey et al., 2000; Rahman et al., 2001; Wen et al., 2001; Fu and Qian, 2002; Abe et al., 2003) cubic (Craft et al., 1996; Shih et al., 1997; Lien and Leschziner, 1996; Wallin and Johansson, 2000; Abdon and Sundén, 2001; Merci et al., 2001; Palaniswami et al., 2001; Song et al., 2001), or even quartic – though incomplete (Craft et al., 1995; Wallin and Johansson, 2000), constitutive relationships for the Reynolds stress. In contrast practically all the proposed NLEDMs (Launder, 1988; Hanjalić, 1994; Abe and Suga, 2001; Knoell and Taulbee, 2001; Park and Sung, 2001; Rokni and Gatski, 2001; Rokni and Sundén, 2003; Abe et al., 2003; Nagaoka and Suga, 2003; Suga, 2003) belong to the quadratic family.

Lastly, item (d) refers to the methodology adopted in formulating NLEVMs and NLEDMs. While some authors simply postulate a non-linear expression for turbulent stresses and/or fluxes, and determine the coefficients by imposing appropriate conditions, others recover a non-linear expression by simplifying second-moment closure models. In particular, algebraic stress models (ASMs) assume that the anisotropy tensor is conserved along a streamline (though a different condition has also been proposed (Rumsey et al., 2001) to account for curvature effects), and that the rate of change and diffusion of the anisotropy is linearly related to the rate of change and diffusion of the turbulence kinetic energy. This approach, originally proposed by Rodi (1972, 1976), leads to implicit forms for the Reynolds stresses. The resulting expressions are rather complex, and cannot be strictly classified as NLEVMs; furthermore ASMs are often reported to give convergence problems in numerical solutions. Gatski and Speziale (1993), Speziale (1997) and Girimaji (1996, 2001), by adopting a non-linear form in an appropriate tensor basis, recover an explicit non-linear expression, termed an explicit algebraic stress model (EASM), see also Park and Sung (1995), Gatski and Jongen (2000), Knoell and Taulbee (2001), Fu and Qian (2002), Wallin and Johansson (2002). On the basis of their close relationship to second-moment closures the authors claim these to be more powerful than ordinary NLEVMs. However, it has to be said that a full tensor basis requires five terms; when fewer terms are used, as is usually the case, the resulting model amounts rather to a least-square fit. Furthermore singular expressions can result in some situations, thereby requiring a ‘regularization’ of the expressions for the stresses (Gatski and Speziale, 1993; Gatski and Jongen, 2000), which then depart from those of the parent second-moment model. Other realizability constraints related to EASMs are discussed by Durbin and Petterson-Reif (1999) and Weis and Hutter (2003).

A similar distinction can be drawn for scalar transport between those in which a non-linear expression for the scalar fluxes is simply postulated and those in which a similar form is obtained as a result of simplifications to second-moment closures for the scalar fluxes. Such simplifications lead to implicit algebraic expressions for the latter; again, by adopting an appropriate basis, the model can be expressed in explicit form, sometimes termed an explicit algebraic heat flux model (EAHM) (Weigand et al., 2002; Park et al., 2003). The full basis in this case requires 10 terms and the development of such a full model has not yet been attempted. Thus current models can be considered as least-square fits to full EAHMs. As for the Reynolds stresses, singularities can arise for the scalar fluxes and ‘regularization’ is then required. An alternative widely used expression for the scalar fluxes is the generalized gradient diffusion hypothesis (GGDH), stemming from application of a higher-order model, (Daly and Harlow, 1970) to heat fluxes (Launder, 1988), which also conserves some relation to second-moment models. It has been used extensively (Hanjalić, 1994; Abe and Suga, 2001; Rokni and Gatski, 2001; Suga, 2003); a higher-order version (HOGGDH) has also been proposed (Nagaoka and Suga, 2003).

The near-wall behaviour of non-linear models have been addressed by a number of authors (Knoell and Taulbee, 2001; Merci et al., 2001; Rahman et al., 2001; Rumsey et al., 2001; Abe et al., 2003; Park et al., 2003). Non-linear models have also been extended to deal with high-speed flows, (Palaniswami et al., 2001), two-phase flows (Mashayek and Taulbee, 2002a; Mashayek and Taulbee, 2002b; Zhou and Gu, 2002), buoyant flows (Wen et al., 2001; So et al., 2002), fluids exhibiting very small (such as liquid metals) or large (liquids) Prandtl numbers (Abe and Suga, 2001; Weigand et al., 2002), or even viscoelastic behaviour (Mompean, 2002; Mompean et al., 2003). This serves to emphasise the practical importance currently attached to non-linear constitutive equations. The critical point in the derivation of non-linear models is the determination of the model coefficients. Compliance with experiments, realizability, and criteria borrowed from thermodynamics have been used. Incidentally, the circumstance that the modelled equations can switch their nature from parabolic to hyperbolic due to an inappropriate choice of the model coefficients was recognized by Weigand et al. (2002).

In the present work non-linear constitutive equations, involving quadratic forms, are devised to allow both the Reynolds stresses and scalar fluxes to be determined. A quadratic form is selected in order to depart relatively little from the well-tried standard $k$–$\epsilon$ model and also because, in previous work, the effect of higher-order terms proved to be relatively small (Abdon and Sundén, 2001). As in all similar approaches the present formulation involves a significant number of undetermined parameters. However, it is shown that the imposition of realisability constraints – positivity of the normal stresses and satisfaction of Schwarz's inequality by the shear stresses and compliance with extremum principles for scalar quantities – results in a substantial reduction in the number of free parameters. The remaining free constants and parameters are then determined by recourse to measurements in simple canonical shear flows. The enforcement of realisability constraints on the Reynolds stresses is achieved mainly through consideration of thin shear flows and mixing layers and, while realisable results are not guaranteed under all general strain conditions, this is clearly a prerequisite to ensuring realisability in more complex flows. None of the currently available non-linear eddy diffusivity models (NLEDM) appear to take account of extremum principles, a consequence of which is that the maximum and minimum values of a strictly conserved scalar quantity arising in any steady solution must lie on the boundaries of the solution domain. Satisfaction of this constraint is of paramount importance in many practical applications and a failure to do so can have catastrophic consequences in computations; species mass fractions less than zero and greater than unity can arise and, for heat transfer problems, temperature profiles may violate the second law. In the present paper a condition on the model coefficients is explicitly enforced to ensure compliance with extremum principles. The resulting complete model, termed a non-linear eddy viscosity and diffusivity model (NLEVDM), is also shown to satisfy joint realisability.

Section 2 presents the proposed form of the constitutive relationship for the Reynolds stresses, and discusses the criteria adopted to identify the NLEVM coefficients. These are defined as a function of an appropriate strain parameter, with the aim of preventing the occurrence of unphysical situations. Similarly, Section 3 presents the form of the constitutive relationships for scalar