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Vortex breakdown in a cylinder with a rotating bottom and a flat stress-free surface

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Abstract

Vortex breakdown and transition to time-dependent regimes are investigated in a cylinder ($H/R = 4$) with a rotating disk and a free-surface. The aim of this study is to show how, by changing upstream conditions it is possible to alter on the flow, particularly the vortex breakdown process. The understanding of such effects on vortex breakdown is very useful in the development of a control strategy in order to intensify or remove the phenomenon. The flow dynamics are explored through numerical solution of the three-dimensional Navier–Stokes equations based on high-order spectral approximations. The use of a flat, stress-free model for the air/water interface is shown to be entirely satisfactory at least for moderate Reynolds numbers. A particular interest of these results is to show how the bubble related to the vortex breakdown becomes attached to the free-surface and grows in diameter as the Reynolds number is increased, $Re \geqslant 2900$. Such a phenomenon removes the cylindrical vortex core upstream of the breakdown which is usually included in classical theories based on idealized models of vortex flows. The flow is shown to be unstable to three-dimensional perturbations for sufficiently large rotation rates. The bifurcated state takes the form of a $k = 3$ rotating wave at $Re = 3000$. The existence of the free-surface promotes the onset of periodicity, with a critical Reynolds number about 15% lower than in the case with a rigid cover. Moreover, the successive bifurcations occur over a much shorter range of Reynolds numbers and lead rapidly to a multi-frequency regime with more than five different frequencies. In the unsteady regime, the vortex breakdown is characterized by an elongated, asymmetric recirculation zone, attached to the free-surface and precessing around the axis of the container. By increasing the rotation, the circular stagnation line on the free-surface takes a more irregular form and starts to move around the axis of the cylinder in the same sense as the rotating disk. Finally, our results show that the vertical boundary layer controls both the vortex breakdown process and the transition to unsteadiness.

Keywords: Nanofluid; Heat transfer enhancement; Laminar flow; Nanoparticle

1. Introduction

When increasing the swirl in a rotating flow a point is reached where the adverse pressure gradient along the jet axis cannot be further overcome by the kinetic energy of the fluid particles flowing in the axial direction, and a recirculation flow zone is set up. Although flow reversal does not necessarily occur in helical and turbulent vortex breakdown (see Sarpkaya, 1995), the development of such a recirculation zone and flow reversal can define a vortex breakdown, bringing the vortex breakdown to an internal flow separation (Leibovich, 1978). This definition is currently considered in most cases in confined cavities, in which the vortex breakdown is often assimilated into one or more recirculation bubbles following the pioneering experimental investigations of Escudier (1984).

The motion of a viscous fluid contained in a closed cylinder, with a rotating disk lid which recirculates the flow inside the container, poses an attractive example of confined swirling flow and provides very well controlled conditions, particularly so for numerical studies. The major characteristics of the flow are known to be determined by

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Nomenclature

H, R	height and radius of the cylinder	(r, z)	dimensionless radial and axial coordinates
Ω	rotation speed of the bottom end wall	$\bar{r}$	dimensionless normalized radius in $[-1, 1]$
ν	dynamic viscosity	$V = (u, v, w)$	dimensionless velocity
g	gravitational acceleration	p	dimensionless pressure
$\Gamma = H/R$	aspect ratio	h, e	axial length and maximum radial extension of the vortex breakdown
$Re = \Omega R^2/\nu$	Reynolds number	L	distance along the axis from the centre of the bottom rotating disk to the upstream fixed point of the bubble
$Fr = \Omega^2 R^2/gH$	Froude number		
(r^*, θ, z^*)	dimensional cylindrical coordinates system in the absolute frame of reference		

two dimensionless parameters: the aspect ratio ($\Gamma = H/R$) and the rotational Reynolds number ($Re = \Omega R^2/\nu$), where H and R are respectively, the height and the radius of the cylinder, Ω the angular velocity of the top end wall, and ν the kinematic viscosity of the fluid.

For certain combinations of (Γ, Re), the confined vortex can undergo breakdown. One of the particular interests is that vortex breakdown may be observed under laminar and steady conditions, without the influence of turbulence, contrary to the case of vortex tubes or in trailing vortices. Moreover, this kind of flow provides a very interesting way for studying transition to turbulence in finite-dimensional systems (Sørensen and Christensen, 1995).

The fundamental nature of the flow (axisymmetric or three-dimensional) has been, at least until recently, the subject of considerable controversy. Using a stability analysis Gelfgat et al. (2001) showed that in the range, $1.63 \leqslant \Gamma \leqslant 2.76$, the dominant perturbation mode is axisymmetric ($k = 0$) while outside this range, the instability is non-axisymmetric as supported by three-dimensional computations of Navier–Stokes equations (see Blackburn and Lopez, 2000; Marques and Lopez, 2001; Serre and Bontoux, 2002). In contrast, Spohn et al. (1998) in experiments and Sotiropoulos and Ventikos (2001) in computations clearly established the asymmetry of the steady vortex bubble at the downstream end. Sotiropoulos and Ventikos (2001) reproduced Lagrangian images of the breakdown bubbles in remarkable agreement with the visualizations photographs of Spohn et al. (1998). What was observed by Sotiropoulos and Ventikos (2001) from Eulerian comparison measures, is that the asymmetry mode in the three-dimensional flow field may be very small but it could nevertheless have a large impact on the Lagrangian dynamics of an axisymmetric flow. Such finding could explain the discrepancy observed between the manifold structures of axisymmetric vortex breakdown in steady regimes observed from numerical integrations of the three-dimensional Navier–Stokes equations (Blackburn and Lopez, 2000; Marques and Lopez, 2001; Serre and Bontoux, 2002) and those of the recent experiments of Spohn et al. (1998) and computations of Sotiropoulos and Ventikos (2001). The topology of the steady vortex breakdown has been very recently specified numerically by Sotiropoulos et al. (2001) and experimentally by Sotiropoulos et al. (2002), and chaotic particle paths were also emphasized within the bubble.

In spite of numerous studies, all the details concerning the question of the symmetry breaking have not been explained. The symmetry breaking could be related to the centrifugal instability, leading an asymmetric flow separation on the container wall as observed experimentally by Spohn et al. (1998) and numerically by Sotiropoulos and Ventikos (1998, 2001). Nevertheless, Sotiropoulos and Ventikos (1998, 2001) showed that in their computations the separation is forced by the distorted structure of their Cartesian numerical grid. In contrast, for Blackburn and Lopez (2000), and Marques and Lopez (2001) this symmetry breaking of the flow is attributed to an inflectional instability of the swirling jet produced by the turning of the Ekman layer on the stationary vertical sidewall. This result looks to be confirmed by a recent stability analysis of Blackburn (2002) carried out in a cavity of aspect ratio $\Gamma = 2.5$ and at $Re = 4000$.

In all studies mentioned above, the rigid end walls impose the no-slip condition on the flow and thus force the formation of three-dimensional boundary layers, which interact with the swirling motion in the inner part of the container. In this way, the characteristics of the vortex flow are determined by the boundary conditions, and by changing them it is possible to act on vortex breakdown conditions. Following this idea, a number of boundary conditions variations have been recently studied: the flat rotating bottom cover has been replaced by a cone (Pereira and Sousa, 1999), or a rod has also been added at the axis (Mullin et al., 2000). The aim of the present study is to investigate the consequence on the flow structure and on the conditions of vortex breakdown, when the upper boundary no-slip condition is replaced by a flat free-surface. Studies of rotating flows in containers with a free-surface are scarce. Experimentally (Spohn et al., 1993) the flow structure including vortex breakdown has been considered in a container of aspect ratio $\Gamma \leqslant 4$ and $Re \leqslant 4000$ and more recently in a $\Gamma = 2$-cylinder (Hirsa et al., 2002). Spohn et al. (1993) particularly emphasized breakdown bubbles attached at the free-surface in steady regimes. Indeed, these authors observed that the use of water in experiments made the flow very sensitive to perturbation