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Nonlinear stochastic estimation of wall models for LES

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Abstract

A key technology for large eddy simulation (LES) of complex flows is an appropriate wall modeling strategy. In this paper we apply for the first time a fully nonparametric procedure for the estimation of generalized additive models (GAM) by conditional statistics. As a database, we use DNS and wall-resolved LES data of plane channel flow for Reynolds numbers, $Re = 2800$, 4000 (DNS) and 10,935, 22,776 (LES). The statistical method applied is a quantitative tool for the identification of important model terms, allowing for an identification of some of the near-wall physics. The results are given as nonparametric functions which cannot be attained by other methods. We investigated a generalized model which includes Schumann's and Piomelli et al.'s model. A strong influence of the pressure gradient in the viscous sublayer is found; for larger wall distances the spanwise pressure gradient even dominates the $\tau_{w,zy}$ component. The first a posteriori LES results are given.

Keywords: Large eddy simulation; Nonlinear data analysis; Computational fluid dynamics; Wall models

1. Introduction

Large eddy simulation (LES) is widely considered as a promising tool for the numerical prediction of transitional and turbulent flows. One of the most urgent problems in the development of LES is the treatment of the near-wall region. There, an exclusively large number of grid points, comparable to direct numerical simulation (DNS), is needed to resolve the flow with classical no-slip and impermeability boundary conditions. The currently available subgrid scale (SGS) models are not able to capture the flow dynamics in the near-wall region with the desired accuracy.

A wall model typically reproduces the logarithmic law of the wall encountered in boundary layers without or with mild pressure gradients, hence the first-order statistics are resolved. Higher-order statistics are either not resolved or included by heuristic assumptions. Difficulties to build a reasonable wall model concern the filter width of LES which should decrease close to the wall, the strong inhomogeneity and anisotropy of near-wall turbulence and lacking theory about the statistics of the involved quantities close to the wall. The filtering problem has been discussed in the literature (Ghosal and Moin, 1995; Vasilyev et al., 1998). It is agreed that typically too large filter widths are applied to resolve the local turbulent small-scales. Statistical approaches fail in the wall region due to the entanglement of the physical processes. Some attempts aim at using symmetry decomposition techniques to obtain local scaling laws in the wall region or similarity laws (Benzi et al., 2002; Gualtieri et al., 2002). Investigations on the change of structure functions with the wall-normal coordinate show that an understanding of intermittency and coherent structures is very important (Toschi et al., 1999).

A tool to obtain information about dependencies of flow quantities is the estimation of conditional statistics on the basis of experimental or numerical data (Adrian, 1979; Langford and Moser, 1999). High-dimensional nonadditive and nonlinear models are hard to interpret and require a huge number of observations for good estimation. Models of algebraic character are limited to second-order (Adrian et al., 1989; Nicoud et al., 2001), usually linear

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stochastic estimation (LSE) being used. This might be misleading if the relations involved are not of quadratic order or not of polynomial form. In particular, highly nonlinear models cannot be captured by this way.

To overcome this restriction and for the sake of generality, we propose to use generalized additive models (GAM) and nonlinear, nonparametric regression (Hastie and Tibshirani, 1990) as a generalization of previous techniques. As a result, one obtains equations involving nonparametric, possibly nonlinear functions in a GAM. To indicate the more general character of our approach, we choose to call it nonlinear stochastic estimation (NLSE).

In this publication, we show how to find GAMs by estimating conditional probabilities for use in nonlinear LES wall models. As a data base, we use DNS and wall-resolved LES data for a plane channel flow at low and moderate Reynolds numbers $Re = 2800$, $Re = 4000$ (DNS), and $Re = 10{,}935$, $Re = 22{,}776$ (LES) with $Re = U_b\delta_{Ch}/\nu$, based on the integral scales U_b and δ_{Ch}, the bulk velocity and the channel half-width, respectively. This corresponds to a Re_τ range of $180 \leqslant Re_\tau \leqslant 1056$. We use known models as a priori test cases which are then generalized and extended to a GAM. A quantitative comparison of the results is possible for our analysis. Filtering has not been investigated due to the known problems of the choice of the filter width. However, in principle results for filtered quantities can be constructed from the unfiltered ones. Using instantaneous, unfiltered values, results in terms of correlations look much worse than for filtered ones, because of the integrative effect of the spatial filter. Thus, good results in this article are improved for filtered variables which are applied in LES. At the end of the paper a few a posteriori LES results of the investigated GAM are presented. The prediction is quite acceptable, especially for the $\tau_{w,zy}$ component.

It has to be mentioned that wall models in LES with their coarse resolution of the near-wall region possess numerical and modeling errors. This plays an important role for the overall accuracy of LES predictions (Breuer, 1998, 2002; Moin, 2002). However, within this paper we mainly concentrate on the derivation of appropriate models since this important topic can only be addressed implicitly, based on a posteriori testing of the improved models (see Section 4.3). Additionally, it is clear that wall models can only be applied if a correlation exists between the wall shear stress and the near-wall velocity. Hence, wall functions in general are restricted to y^+ values of the order O(50). Nevertheless, that leads to a decrease in wall-normal grid resolution of two orders of magnitude compared with the no-slip boundary condition and additionally allows to increase the time step size. Thus wall functions are able to substantially reduce the computational costs of LES as will be particularized below.

In general, the LES technique is based on the idea that only the large scales are computed directly which allows to use a coarser grid compared to the resolution required for DNS (Breuer, 2002; Moin, 2002; Pope, 2000). The resulting subgrid scale contributions have to be modeled. The procedure is formulated by the filtered Navier–Stokes equations here given for an incompressible fluid

$$\partial_j \overline{u}_j = 0, \tag{1}$$

$$\partial_t \overline{u}_i + \partial_j(\overline{u}_j\overline{u}_i) = \frac{1}{Re}\partial_{jj}\overline{u}_i - \partial_i\overline{p} - \partial_j\tau_{ij}. \tag{2}$$

The quantity τ_{ij} represents the subgrid scale stress tensor, $\overline{u}, \overline{v}, \overline{w}$, and $\overline{p}$ denote the filtered streamwise, wall-normal and spanwise velocity components, and the pressure. The respective coordinates are x, y, and z. The summation convention is used.

The quality of LES results strongly depends on the quality of the subgrid scale model for τ_{ij}. It turned out that reasonable models can be developed for the bulk flow, where the assumption of nearly isotropic turbulence at least approximately holds. Simulations with a coarse grid yield, however, insufficient results close to solid boundaries where the flow is typically inhomogeneous and anisotropic. Attempts to fully resolve the near-wall region lead to very expensive runs because the largest part of the simulation is occupied with the near-wall region. Piomelli and Balaras (2002) estimated the computational costs of a wall-resolved LES to scale as $Re^{2.4}$. Hence appropriate wall models reducing the computational costs to scale as $Re^{0.5}$ are urgently needed to avoid the expensive DNS-like resolution for the near-wall region (see the review by Piomelli and Balaras (2002)).

In the following those models are briefly described which are relevant in the further analysis. We focus on the algebraic models by Schumann (1975), Piomelli et al. (1989), the ejection model (Piomelli et al., 1989), and the gradient model. The known models are briefly explained and then generalized. Other modifications of Schumann's model (Balaras et al., 1996; Grötzbach, 1987; Hoffmann and Benocci, 1995; Piomelli et al., 1989; Werner and Wengle, 1993) will not be considered.

The remaining sections of the paper are organized as follows: in the next section, the nonparametric generalization of wall models is explained and some details of the stochastic estimation procedure are given. In Section 3 the generation and pre-processing of the DNS and LES data involved is explained. Finally, the results for the models investigated are presented and discussed in Sections 4 and 5.

2. Nonlinear stochastic estimation and generalized additive models

In this section, we will generalize existing wall functions towards a nonparametric representation. In the approach of Schumann (1975) a phase coincidence of the instantaneous wall shear stress and the tangential velocity component in the first off-the-wall cell is assumed. Combining this assumption with the impermeability condition, the resulting system of equations for a wall parallel to the x-axis reads