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Measurements of velocity–acceleration statistics in turbulent boundary layers

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Abstract

An advanced laser Doppler velocimetry system is developed to acquire measurements of fluctuating velocity–acceleration statistics in turbulent boundary layers. These correlations are enabled by customized burst signal processing that estimates both the Doppler frequency and the rate-of-change of Doppler frequency, which are related to the particle velocity and acceleration by the interference fringe spacing. The measurements give important insight into the near-wall turbulence structure since the statistical correlations of interest, $\overline{u_i a_j}$ appear directly in the Reynolds stress transport equations as a sum of the velocity–pressure gradient correlation, $-\frac{1}{\rho}\overline{\left(u_i\frac{\partial p}{\partial x_j}+u_j\frac{\partial p}{\partial x_i}\right)}$, the dissipation rate, $2\nu\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}}$, and the viscous diffusion, $\nu\nabla^2\overline{u_i u_j}$. The immediate power of such measurements is that combinations of terms in the Reynolds stress transport equation may be characterized by a single statistical measurement at one location in the flow—no gradients need be computed. In the present paper, data are presented for a constant-pressure 2D turbulent boundary layer at $Re_\theta = 6800$. Near-wall results for the dominant term in the velocity–acceleration tensor, the streamwise correlation $\overline{ua_x}$, compare favorably with DNS for the same quantity at $Re_\theta = 1410$ and $Re_\tau = 640$; furthermore, the quantity exhibits no Reynolds number effects within experimental uncertainties. The balance of the velocity–acceleration equation in the streamwise direction is presented, giving the first measurements for the profile of the velocity–pressure gradient correlation with this technique. This study exhibits the potential of the technique to be applied to more complex flows, particularly those 3D separating flows in which the motions contributing to the velocity–acceleration correlations become dominant.

Keywords: Turbulent acceleration; Flow acceleration measurement; Laser Doppler velocimetry/anemometry; Turbulent boundary layers; Reynolds stress transport; Turbulence structure

1. Introduction

In this paper, measurements of velocity–acceleration correlations obtained from an advanced laser Doppler velocimetry (LDV) system are presented. The principle of the simultaneous velocity and acceleration measurements is that for a static interference fringe system, the frequency of particle fringe crossings is proportional to particle velocity, while the first time derivative of this frequency is proportional to the particle acceleration. The proportionality constant is the fringe spacing. By utilizing customized signal processing techniques both the Doppler frequency and the rate of change of Doppler frequency (or chirp rate) may be determined.

Interest in Lagrangian acceleration measurement has been growing with the advent of some new optical particle tracking technologies and the increased computational and storage capacities of modern computers and digital signal processors. Due to the complexity of the measurements, very little information exists about the acceleration structure in turbulent flows. Published techniques include indirect measurement via the isotropy assumption by measuring the fourth-order velocity structure functions (Hill and Thoroddsen, 1997), as well as direct studies using DNS (Vedula and Yeung, 1999), particle tracking velocimetry techniques (Virant and Dracos, 1997; LaPorta et al.,

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2001; Voth et al., 1998, 2002), particle image velocimetry (PIV) (Christensen and Adrian, 2002), and LDV (Lehmann et al., 2002). In the current study, LDV is chosen primarily due to its exceptional resolution in the near-wall region.

Previous work has shown the potential for estimating instantaneous particle accelerations using LDV. The differential LDV technique can be directly extended to make acceleration measurements by simply adjusting the signal processing. In work reported by Lehmann et al. (2002), the authors compared three signal processing methods for estimating particle accelerations and used one of the techniques in a flow situation. The results validated that LDV could successfully be extended to acquire acceleration measurements in turbulent flows.

Of particular interest in the current study is the role of the correlation between the fluctuating velocity and fluctuating acceleration in the Reynolds stresses transport (RST) equations. This term is chosen for two reasons, first because it appears directly in the RST equations as a combination of up-to-now difficult to measure terms. Second because the correlation results in low uncertainties relative to the individual uncertainties of the velocities and the accelerations, since the random noise content will not result in any net correlation.

The relationship between the velocity–acceleration correlation and the Reynolds stress transport may be seen through an analysis of the Navier–Stokes (N–S) equations. The conventional notations for Reynolds decomposition are used to follow—uppercase variables are instantaneous quantities while lowercase variables denote fluctuating quantities and over lines denote the average of the quantity beneath. A basic, *linear* form of the N–S equations in tensor notation is

$$A_i = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \nu\nabla^2 U_i, \tag{1}$$

where A_i is the Lagrangian fluid particle acceleration, ρ is the fluid density, P is the pressure, and U_i is the particle velocity. Since this equation is linear, the fluctuating form is analogous. By multiplying the fluctuating form of Eq. (1) by the fluctuating velocity u_j and Reynolds averaging one obtains

$$\overline{a_i u_j} = -\frac{1}{\rho}\overline{u_j\frac{\partial p}{\partial x_i}} + \overline{\nu u_j \nabla^2 u_i}. \tag{2}$$

By switching the indices in Eq. (2) and adding the result back with the original equation, the following form results:

$$\overline{a_i u_j} + \overline{a_j u_i} = -\frac{1}{\rho}\overline{u_j\frac{\partial p}{\partial x_i} + u_i\frac{\partial p}{\partial x_j}} + \overline{\nu u_j\nabla^2 u_i + u_i\nabla^2 u_j}. \tag{3}$$

The final term on the right hand side of this equation may be decomposed into dissipative and diffusive terms, as shown by Pope (2000):

$$\overline{\nu u_j\nabla^2 u_i + u_i\nabla^2 u_j} = -2\nu\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}} + \nu\nabla^2\overline{u_i u_j}, \tag{4}$$

which appear directly in the RST equations given as

$$\frac{\partial\overline{u_i u_j}}{\partial t} + U_k\frac{\partial\overline{u_i u_j}}{\partial x_k} + \frac{\partial\overline{u_i u_j u_k}}{\partial x_k} = -\overline{u_i u_k}\frac{\partial U_j}{\partial x_k} - \overline{u_j u_k}\frac{\partial U_i}{\partial x_k} - \frac{1}{\rho}\overline{\left(u_i\frac{\partial p}{\partial x_j} + u_j\frac{\partial p}{\partial x_i}\right)} - 2\nu\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}} + \nu\nabla^2\overline{u_i u_j}. \tag{5}$$

In this way, the velocity–acceleration tensor is directly related to the RST equations by the sum of the velocity–pressure gradient correlation, $\Pi_{ij} = -\frac{1}{\rho}\overline{\left(u_i\frac{\partial p}{\partial x_j} + u_j\frac{\partial p}{\partial x_i}\right)}$, viscous diffusion, $D_{\nu ij} = \nu\nabla^2\overline{u_i u_j}$, and dissipation rate, $\epsilon_{ij} = 2\nu\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}}$ tensors. Thus the velocity–acceleration fluctuation correlation measurements give an alternate equation (apart from Eq. (5) itself) for determining Π_{ij} when measurements for $D_{\nu ij}$ and ϵ_{ij} are possible.

The importance of the velocity–pressure gradient term in complex wall-bounded flows has been shown for low Reynolds numbers by the DNS of Coleman et al. (2000). In the strained channel flow DNS, the authors discovered that Π_{ij} is of primary importance to the evolution of the Reynolds stresses. They showed that the lag between the mean shear rate and the Reynolds shear stresses, a key problem in 3D flows, is primarily due to this term.

In this paper, we wish to utilize the capabilities of a new advanced LDV for velocity–acceleration correlation measurements in a 2D constant pressure turbulent boundary layer at $Re_\theta = 6800$. This simplest of turbulent boundary layer flows has been the subject of many experimental studies in the past. DeGraaff and Eaton (2000) give an extensive review of the work that to this point has primarily involved the measurement of velocity statistics. The velocity–acceleration measurements add to the database of information on this flow and give insight into the mechanisms leading to Reynolds stress transport.

2. Apparatus and instrumentation

Measurements were taken in the turbulent boundary layer of the Aerospace and Ocean Engineering Department small boundary layer wind tunnel. A detailed description of this facility in its present configuration is given by Bennington (2004). The nominal dimensions of the test section are 23 cm wide by 10 cm high by 2 m long. The measurements were acquired 1.16 m downstream of the contraction on the centerline of the tunnel. The floor boundary layer was tripped to turbulence using a pair of square bars with edges of 0.32 cm spanning the width of tunnel floor. The two bars were spaced by 20 cm with 20 grit sand paper attached to the floor between the bars. The measurements were acquired at over 350 bar-heights downstream of the trip arrangement, resulting in a fully relaxed boundary layer. The current measurements showed the inviscid core of the wind tunnel to have a velocity of 26.9 m/s with 0.3% turbulence intensity and another 0.7% low frequency unsteadiness. The unsteadiness was found to be low frequency, below 10 Hz and thus did not correlate with the higher-frequency active turbulent motions in the boundary layer.