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Form drag effect on the onset of non-linear convection and Hopf bifurcation in binary fluid saturating a tall porous cavity

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A B S T R A C T

This paper reports a numerical study of natural convection in at all porous enclosure filled with a binary fluid. The Darcy–Dupuis model, which includes effects of the form drag force, is adopted to describe the flow in the porous medium. The two vertical walls of the cavity are subject to constant gradients of temperature while the two horizontal ones are kept adiabatic and impermeable. Concentration gradients are assumed to be induced either by the imposition of constant gradients of solute on the vertical walls of the system ($a = 0$; double diffusive convection) or by the Soret effect ($a = 1$). Governing parameters of the problem under study are the thermal Rayleigh number R_T, form drag parameter G, buoyancy ratio φ, Lewis number. Le, normalized porosity ε, and aspect ratio of the cavity A. The case of equal and opposing thermal and solutal buoyancy forces, $\varphi = -1$, is considered. For this situation, an equilibrium solution corresponding to the rest state is possible and the resulting onset of motion can be either supercritical or subcritical. A semi-analytical solution, valid for an infinite layer ($A \gg 1$) assuming parallel flow, is derived. Based on the linear stability theory, the onset of motion from the rest state is predicted for both double diffusive and Soret convection. The onset of Hopf bifurcation, characterizing the transition from a convective steady state to oscillatory state, is also studied. The influence of the governing parameters on the onset of motion and the resulting fluid flow, temperature and concentration fields is discussed in detail. The existence of supercritical, subcritical and oscillatory convective modes is demonstrated. A good agreement is found between the predictions of the parallel flow approximation and the numerical results obtained by solving the full governing equations. The existence of multiple solutions and traveling waves for a given set of the governing parameter is demonstrated and leads to the existence of a bistability phenomenon. Overall, the form drag behaves as a stabilizing effect and is seen to affect considerably the onset of subcritical convection and Hopf bifurcation.

1. Introduction

Recently, natural convection of binary fluids in porous enclosures has become increasingly a subject of intensive research, in view of its wide range of applications in many engineering problems. This is due to the fact that this type of fluid can give rise to a large variety of flow behaviors, resulting from the coupling between the temperature and the concentration fields, which cannot occur in a single component fluid. Two types of problems are possible in regard to the solutal contribution to the total buoyancy force induced in the fluid mixture by both the thermal and solutal gradients. In double diffusive (thermosolutal or thermohaline) problems, the thermal and solutal gradients are both externally imposed on the system. In Soret (thermal-diffusion) induced convection, when a temperature gradient is applied to a binary mixture initially homogeneous, thermal diffusion takes place giving rise to a solutal gradient. Comprehensive reviews of studies related to this topic have been reported in references [1–6].

The earlier works on natural convection of binary fluids are concerned with the onset of motion in a horizontal porous layer, subject to vertical temperature and concentration gradients [7–11]. On the basis of the linear stability theory, criteria for the onset of convection via stationary and oscillatory modes were derived by these authors for various thermal and solutal boundary conditions. Also, it is well known that the onset of motion, in a binary fluid layer, can occur at finite amplitude, i.e. be subcritical. This type of bifurcation is possible provided that the buoyancy forces are opposing each other and when the Lewis number is greater than unity. This

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Nomenclature

A	aspect ratio of the enclosure, $A = H'/L'$
a	real number, $a = 0(1)$ for double diffusion (Soret)
C_S	dimensionless concentration gradient in y-direction
C_T	dimensionless temperature gradient in y-direction
D	isothermal diffusion coefficient, $m^2\ s^{-1}$
D'	thermos-diffusion coefficient, $m^2\ s^{-1}\ K^{-1}$
g	gravitational acceleration, ms^{-2}
G	form drag parameter, $\tilde{K}\alpha/\upsilon L'$
H'	height of the enclosure, m
j'	constant solute flux per unit area, $kg\ m^{-2}\ s^{-1}$
k	thermal conductivity, $W\ m^{-1}\ K^{-1}$
K	permeability, m^2
$\tilde{K}$	material parameter, m
L'	width of the enclosure, m
Le	Lewis number, α/D
N	species fraction of the reference component
N_0	initial species fraction of the reference component
Nu	Nusselt number, Eq. (15)
p	dimensionless pressure
q'	constant heat flux per unit area, $W\ m^{-2}$
R_T	Rayleigh number, $\rho g \beta'_T \Delta T' L'/\alpha\upsilon$
S	normalized species fraction, $N/\Delta N$
Sh	Sherwood number, Eq. (15)
ΔN	characteristic species fraction difference of the reference component
t	dimensionless time
T	dimensionless temperature
$\Delta T'$	characteristic temperature difference, $q'L'/k$
u	dimensionless velocity x-component
v	dimensionless velocity y-component
x, y	cartesian coordinates

Greek symbols

α	fluid thermal diffusivity, $m^2\ s^{-1}$
β_N	species expansion coefficient
β'_T	thermal expansion coefficient, K^{-1}
ε	normalized porosity of the porous medium, ϕ/σ
ρ	density of fluid, $kg\ m^{-3}$
φ	buoyancy ratio, $\beta_N \Delta N/\beta'_T \Delta T'$
μ	dynamical viscosity, $kg\ m^{-1}\ s^{-1}$
υ	kinematic viscosity, $m^2\ s^{-1}$
σ	heat capacity ratio, $(\rho C)_p/(\rho C)_f$
ϕ	porosity of the porous medium
Ψ	dimensionless stream function, Ψ'/α

Superscript

$'$	dimensional quantities

Subscript

o	refers to the value taken at the centre of the cavity

situation has been investigated recently, both analytically and numerically, by Mamou and Vasseur [12] and Bahloul et al. [9], among others. The critical Rayleigh numbers for the onset of subcritical convection, for both double diffusive and Soret induced convection, were derived on the basis of the parallel flow approximation. Explicit expressions for velocity, temperature and solute distributions in the core of the layer have been obtained in terms of the governing parameters of the problem.

The onset of convection in a vertical porous layer subject to horizontal temperature and concentration gradients has also be investigated for the particular case when the thermal and solutal buoyancy forces are of opposite sign and equal in magnitude. For this situation, it has been demonstrated by Trevisan and Bejan [13] that, when the Lewis number is equal to unity, the temperature and concentration fields are identical such that the only possible solution is that corresponding to the rest state. However, even though the pure diffusive regime is also an exact solution when the Lewis number is not equal to unity, there is no reason to expect that it will remain unconditionally stable. Double diffusive convection instability in a vertical porous enclosure subject to constant horizontal fluxes of heat and solute has been studied by Mamou et al. [14]. On the basis of the linear stability theory, the marginal states of instability via stationary and oscillatory modes have been determined in terms of the Lewis number, normalized porosity of the porous medium and the enclosure aspect ratio. Also, an analytical solution valid for a tall enclosure has been derived on the basis of the parallel flow approximation. The resulting nonlinear model indicates that the onset of finite amplitude convection occurs at a subcritical Rayleigh number, which is function of the Lewis number. The same problem was reconsidered by Mamou et al. [15] upon considering a porous medium modeled according to the Brinkman-extended Darcy's law. Numerical results, based on the Galerking and finite element methods, were obtained for different types of thermal and solutal boundary conditions and aspect ratios of the cavity. The transition from the Darcy (densely packed) porous medium to the clear fluid is discussed. Amahmid et al. [16] investigated analytically the double diffusive convection in a Brinkman layer. A closed form solution was derived to predict the velocity, temperature and concentration fields in terms of the governing parameters of the problem. The subcritical Rayleigh numbers, above which a unicellular convective cell exists, were predicted in terms of the Lewis and Darcy numbers. Double diffusive convection in an inclined porous enclosure has been studied by Mamou et al. [17]. Criteria for the onset of convection from the rest state have been derived numerically. The existence of subcritical convection was also demonstrated. The same problem was reconsidered by Karimi-Fard et al. [18] for a cavity with the opposite walls maintained at different but uniform temperature and concentration. Numerical confirmation of the onset of motion predicted by the linear stability theory was obtained for a square cavity. Also, the numerical simulations show the existence of multiple solutions.

The first study concerning the onset of Soret driven convection in a vertical cavity subject to a horizontal temperature gradient, for the condition of equal and opposite thermal and solutal buoyancy forces, seems to be due to Marcoux et al. [19]. The supercritical Rayleigh number for the onset of convection from the rest state has been predicted on the basis of the linear stability theory, in term of the aspect ratio of the cavity. Stability analysis of thermosolutal convection in a vertical porous enclosure subject to different thermal and solutal boundary has been performed by Mamou [20]. The linearized governing equations were solved numerically using a finite element method. It was reported that an increase in the porosity of the porous media delays the appearance of oscillatory flows. Also, relying on the linear stability theory, the stability of small perturbations from the rest state has been studied numerically by Joly et al. [21,22] for the case of Soret induced convection. It was found that the dependence of the supercritical Rayleigh number on the Lewis number is different from that of the double-diffusive problem. The critical Rayleigh number for the onset of subcritical convection was also obtained analytically. For finite amplitude convection, explicit expressions