International Journal of Heat and Mass Transfer 100 (2016) 362–377

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Transient heat conduction problem with radiation boundary condition of statistically inhomogeneous materials by second-order two-scale method

Zhiqiang Yang [a,*], Junzhi Cui [b], Yi Sun [a]

[a] Department of Astronautic Science and Mechanics, Harbin Institute of Technology, Harbin 150001, China
[b] LSEC, ICMSEC, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China

ARTICLE INFO

Article history:
Received 23 December 2015
Received in revised form 21 March 2016
Accepted 25 April 2016
Available online 10 May 2016

Keywords:
SSOTS method
Radiation boundary condition
Heat transfer
Inhomogeneous porous materials
Random distribution

ABSTRACT

This paper develops a statistical second-order two-scale (SSOTS) method for predicting transient heat transfer performance of statistically inhomogeneous porous materials. In these materials, the complicated microscopic information of inclusions, including their shape, size, orientation, spatial distribution, volume fraction and so on, leads to changes of the macroscopic thermal properties. Also, heat radiation effect at microscale has an important impact on the macroscopic temperature field. A novel unified homogenization procedure, based on two-scale asymptotic expressions depending on the position variables, has been established. The second-order two-scale formulations for computing the transient heat transfer problem are given successively. Then, the statistical prediction algorithm based on the proposed two-scale model is described in details. Finally, numerical examples for porous materials with varying probability distribution models are calculated by the algorithm given, and compared with the data by the theoretical results. The comparison shows that the statistical two-scale computational method developed is useful for determination of the heat transfer properties of porous materials and demonstrates its potential applications in actual engineering computation.

1. Introduction

Porous materials are widely used in a variety of engineering and industrial products due to their growing importance in functional material design, excellent high temperature resistance, together with light weight and high radiation attenuation coefficient compared to other traditional materials. Specially, with rapid development of aerospace industry, porous materials designed as heat insulator for thermal protection system (TPS) have attracted tremendous attention and wide research interests [1–7]. According to the microscopic characterization of inclusions, random composites can be divided into statistically homogeneous and inhomogeneous composite materials [8–10], shown in Fig. 1. As for statistically inhomogeneous materials, the shape, orientation, size, spatial distribution and volume fractions of inclusions gradually change with position, such as functionally graded materials (FGMs) (Fig. 1(b) [8]) by varying the volume fractions [9,10], cancellous bones by varying pore sizes [11], and polymers by varying fiber orientations [12]. The microstructures that the inclusion dispersions gradually vary result in changes of the macroscopic physical and mechanical properties of the materials. Inevitably, these kinds of statistically inhomogeneous materials have attracted a great deal of attention from scientists and engineers [5,8–12].

Heat transfer in porous materials contains three ways: conduction, convection and radiation. Convection takes place by flow of fluids, and can be neglected at low pressure or in closed-cell porous materials [1,2]. Radiation is a way of heat transmission and plays a significant factor in modern technology, especially as the temperature on a visible surface of the system is high enough. Some interesting work has been contributed to consider the interior surface radiation of porous materials in the past years. Liu and Zhang [1] gave the effective macroscopic thermal properties of the coupled conduction and radiation problem with small parameter ε. Bakhvalov [3] introduced the asymptotic homogenization method for the solution of those problems. On this basis, Cui et al. [4,5] studied the conduction and radiation heat transfer problem with rapidly oscillating coefficients, and obtained a high-order expansion of the solution for the problem. Later, the high-order multiscale method is also developed to predict the thermal–mechanical coupling properties of porous materials with surface radiation [13,14].

* Corresponding author at: No. 92 West Dazhi Street, Nan Gang District, Harbin City, Heilongjiang Province 150001, China. Tel./fax: +86 451 86418124.
E-mail address: yangzhiqiang@hit.edu.cn (Z. Yang).

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2016.04.093
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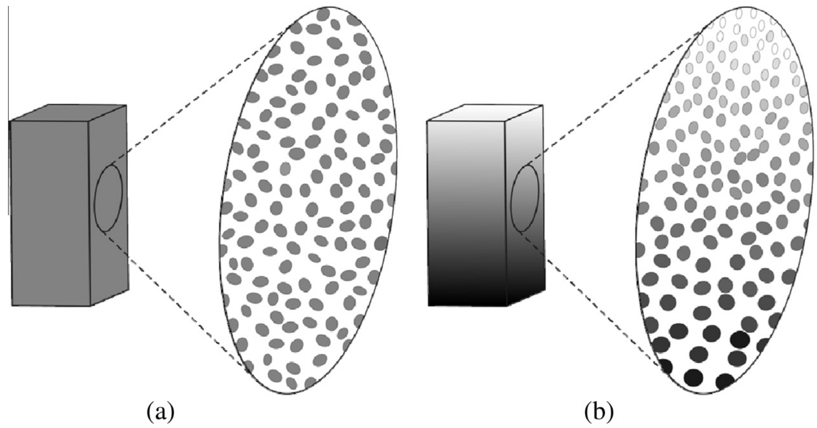

Fig. 1. Random composites (a): statistically homogeneous material; and (b): statistically inhomogeneous material.

Allaire and El Ganaoui [6] and Ma and Cui [7] discussed the heat conduction equation with non-classical radiation boundary conditions by asymptotic expansion homogenization method, and justified the convergence. Yang et al. [5] developed a statistical two-scale method for the heat conduction problem of statistically homogeneous porous materials with interior surface radiation, and gave detailed algorithm procedure.

Solving the heat conduction problem with radiation boundary condition which arises in statistically inhomogeneous porous materials will be discussed in this paper. Particular attention is focused on the treatment of this materials that have the complex and varied microstructure. Under such condition, all microscale heterogeneities may be not practicable even using the advanced supercomputers owing to the requisite of tremendous amount of computer memory and CPU time. In order to solve this kinds of problem, homogenization method and the corresponding multiscale algorithms, which are described in sources [13–17] and other references, are introduced. It includes two different scales in the analysis and computation process, one associated with the macroscopic homogeneous part and the other associated with inhomogeneous microstructure part, i.e. a macroscale and a microscale. By introducing two length scales, one can effectively analyze sophisticated functionally graded components with complex microstructures, which cannot only save the computational resources but also ensure the calculation accuracy. For the physics field problems of composite materials with stationary random distribution, Jikov et al. [18] proved the existences of the homogenization coefficients and the homogenization solution. On the basis, Cui et al. established a statistical second-order two-scale analysis method by introducing a random sample model to predict physical or mechanical properties of the inhomogeneous composite structure [19–24]. The algorithm for generation of the random microstructure was developed by Yu et al. [20], which is based on a model for the probability distribution of the grains (pores). Compared with the other micromechanics method, such as Mori–Tanaka method [25], self-consistent method [26], generalized self-consistent method [27] and Maxwell–Eucken model [28], the homogenization analysis method has a rigorous mathematical background and it can contain almost all the microcosmic structure information of the composites. Goupee and Vel [29] proposed a class of algorithms for the multiscale thermoelastic analysis of random heterogeneous materials and demonstrated the approach on two multiscale problems, a functionally graded Al/Al_2O_3 beam and W/Cu component with bi-directional grading. Meanwhile, Goupee and Vel [30] created a multiscale framework for analyzing the thermo-mechanical performance of functionally graded composites at both macroscopic and microscopic length scales. From these works, it may be concluded that homogenization techniques allow sufficiently accurate predictions of effective macroscopic properties of arbitrarily complex microstructures. Moreover, such techniques permit calculation of the stress and temperature fields at the microscopic scale.

Different from the previous works, the aim of this paper is to determine equivalent material parameters and effectively predict the heat transfer properties of statistically inhomogeneous porous materials. By a constructive way, we introduce the higher-order correction terms into the asymptotic expansion of the temperature field, and then obtain a family of cell problems. A novel statistical two-scale analysis method with less effort and computational cost is developed for computing the transient heat conduction problem with nonlinear radiation boundary condition. Finally, some typical examples are given to show the accuracy and efficiency of the method developed.

This paper is organized as follows. In the following section, the microscopic configurations for statistically inhomogeneous porous materials are represented. Section 3 is devoted to the formulation of the SSOTS method, and the corresponding finite element algorithms are given in details in Section 4. In Section 5, the numerical results for the nonlinear coupled problem obtained by the proposed method, are shown, which demonstrate the statistical two-scale analysis method is effective. Finally, the conclusions are given in Section 6.

Throughout the paper the Einstein summation convention on repeated indices is adopted.

2. Representation of microscopic configurations of statistically inhomogeneous porous materials

Suppose that the investigated porous materials are made from matrix and random cavities. Refer to Ref. [19,20]. All the cavities are considered as ellipsoids, which are randomly distributed in the matrix. From the engineering survey and the statistic fitting method of data, the microstructure of inhomogeneous porous materials is represented as follows:

(1) In the investigated structure Ω^ε, there exists a constant ε satisfying $0 < \varepsilon \ll L$, where L denotes the macroscale of Ω^ε, as shown in Fig. 2(a). If at any point inside Ω^ε there exist cells with the same size ε, and the random distribution model of inclusions is the same in each cell, then the composites are called as statistically homogeneous materials. If at an arbitrary point x_0 inside Ω^ε there exists a cell with size ε_{x_0} and its random distribution model depends on x_0, then the