International Journal of Heat and Mass Transfer 100 (2016) 599–607

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

An analytical method to estimate spatially-varying thermal contact conductances using the reciprocity functional and the integral transform methods: Theory and experimental validation

Ricardo S. Padilha [a], Marcelo J. Colaço [a,*], Helcio R.B. Orlande [a], Luiz A.S. Abreu [b]

[a] Department of Mechanical Engineering (PEM/COPPE), Federal University of Rio de Janeiro (UFRJ), Cx. Postal 68503, Rio de Janeiro, RJ 21941-972, Brazil
[b] Department of Mechanical Engineering and Energy, Rio de Janeiro State University (UERJ/IPRJ), Rua Bonfim, 25, Vila Amélia, Nova Friburgo, RJ 28625-570, Brazil

ARTICLE INFO

Article history:
Received 8 December 2015
Received in revised form 1 April 2016
Accepted 16 April 2016
Available online 13 May 2016

Keywords:
Inverse heat transfer problem (IHTP)
Reciprocity functional
Classical integral transform technique (CITT)
Thermal contact conductance

ABSTRACT

The increasingly interest in using composite materials in engineering application requires the proper knowledge of the interaction that occurs between their layers. One of these interactions is related to the temperature jump and heat flux through the interface of different materials, known as thermal contact resistance, or its reciprocal, the thermal contact conductance. Methods to estimate thermal contact resistance usually require temperature measurements taken inside the sample (test body) and complicated experimental arrangements. In this work we propose an analytical, non-iterative and non-intrusive method to solve an inverse heat transfer problem in order to estimate a one-dimensional steady-state distribution of the thermal contact conductance, combining the reciprocity functional and the Classical Integral Transform Technique (CITT). This paper is an extension of our previous works, where the solution procedure was developed numerically and required the solution of two linear systems. In this paper, the estimate is reduced to a single algebraic equation. The method was applied to some test cases using simulated measurements and results were compared with the exact solution showing a good agreement between them. A validation involving a double-layered material was also conducted, where an infrared camera was used to measure the temperature non-intrusively. A micromachinery produced flaw was created in the contact between the two materials and, once the temperature measurements were available, the method was able to identify the flaw location in 0.2 s using an Intel Atom(TM) CPU N450 1.66 GHz.

1. Introduction

Many processes in engineering and related sciences require the complete knowledge of the interaction that occurs in the interface of materials or living bodies, or even in mixed boundaries, such as a material body and a living organism that are placed in contact. These interfaces usually present some gaps between the materials. One of these interactions is related to the heat transfer process in the contacting interface of two bodies where a temperature discontinuity exists. This temperature jump is the result of a thermal resistance to the heat flux in the interface, been called thermal contact resistance, or its reciprocal, thermal contact conductance-TCC.

In many industrial applications the previous knowledge of the TCC is essential for the design and proper operation of systems and equipment, such as in aerospace [1], electronic [2], nanoparticles [3], refrigeration [4], heat exchangers [5] and aviation engines [6].

Most of existing procedures to estimate TCC are experimental and require very complex techniques and complicated arrangements. In addition, many of these techniques also require the knowledge of the surface profiles of the materials in contact, such as their roughness, and temperature measurements taken inside the materials, close to the contact interface.

Physically, the interface of two materials is characterized by an irregular distribution of micro regions. Only in some parts of these micro regions the materials are in perfect contact. When the materials are submitted to a heat flux, in micro regions where the materials are in contact the temperature across the interface presents continuity, while in micro regions without contact a temperature discontinuity between the contacting surfaces is observed. Considering this, TCC (h_Γ) is defined as the ratio of the heat flux (q_Γ) to the temperature jump (ΔT_Γ) at the interface Γ between the materials:

---

* Corresponding author.
E-mail addresses: rspadilha@hotmail.com (R.S. Padilha), colaco@ufrj.br (M.J. Colaço), helcio@mecanica.ufrj.br (H.R.B. Orlande), abreu.l@gmail.com (L.A.S. Abreu).

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2016.04.052
0017-9310/© 2016 Elsevier Ltd. All rights reserved.

Nomenclature

Abbreviations

CITT	Classical Integral Transform Technique
MCMC	Markov-Chain Monte Carlo
RF	Reciprocity Functional
TCC	Thermal Contact Conductance

Latin symbols

a, b	dimensions of the acrylic sample [m]
Bi	Biot Number
F_1	harmonic test function related to region Ω_1 in the first auxiliary problem [$1/\mathrm{m}^{1/2}$]
F_2	harmonic test function related to region Ω_2 in the first auxiliary problem [$1/\mathrm{m}^{1/2}$]
$\bar{F}$	integral transform of F_1 or F_2 [$\mathrm{m}^{1/2}$]
G_1	harmonic test function related to region Ω_1 in the second auxiliary problem [$1/\mathrm{m}^{1/2}$]
$\bar{G}$	integral transform of G_1 [$\mathrm{m}^{1/2}$]
h	thermal contact conductance [$\mathrm{W}/(\mathrm{m}^2\mathrm{K})$]
k	thermal conductivity [W/(m K)]
$\mathbf{n}$	normal unitary vector pointing outwards the boundary [m]
N	norm of the associated eigenvalue problem for the auxiliary problems [1/m]
N_1	number of terms for the linear expansion in the first auxiliary problem
N_2	number of terms for the linear expansion in the second auxiliary problem
q	heat flux [W/m^2]
T	temperature [K]
x, y	Cartesian coordinates [m]
X	dimensionless variable in the x direction
Y	temperature measurement [K]

Greek symbols

α	linear expansion coefficient in the first auxiliary problem [$(\mathrm{K}^2\ \mathrm{m}^{5/2})/\mathrm{W}$]
β	generic basis function in the first auxiliary problem [$\mathrm{W}/(\mathrm{K}\ \mathrm{m}^{5/2})$]
γ	generic basis function in the second auxiliary problem [$1/\mathrm{m}^{1/2}$]
Γ	interface between the two materials in contact
Γ_1	lateral boundary surface of material 1
Γ_2	lateral boundary surface of material 2
Γ_0	upper boundary surface of material 1
Γ_{00}	bottom boundary surface of material 2
ε	Gaussian variable with zero mean and unity variance
λ	eigenvalue of the associated eigenvalue problem for both auxiliary problems [1/m]
μ	linear expansion coefficient for the second auxiliary problem [$\mathrm{W}/\mathrm{m}^{3/2}$]
σ	standard deviation of the temperature measurements [K]
φ	eigenfunction of the associated eigenvalue problem for both auxiliary problems
Ψ	orthonormal basis functions used in the auxiliary problems [$1/\mathrm{m}^{1/2}$]
Ω	material region
$\mathfrak{R}$	reciprocity functional [$\mathrm{K}/\mathrm{m}^{1/2}$]

Subscripts

0	upper boundary surface of material 1
00	bottom boundary surface of material 2
1	material region 1
2	material region 2
i, j	frequency of the orthonormal basis function
m	number of eigenvalues in the eigenvalue problems

$$h_\Gamma = \frac{q_\Gamma}{\Delta T_\Gamma} \tag{1}$$

From this definition, for regions with perfect contact between the materials, where no temperature discontinuity is verified, TCC is infinity. On the other hand, in micro regions where the materials are not in contact, presenting a temperature discontinuity across the interface, TCC has a finite value. In the limiting case, where a perfect thermal insulation interface is present, TCC is null.

TCC can also be analyzed using two approaches: local or lumped/global. The local one is related to some physical and geometric irregularities in the micro regions between the materials. In these cases, TCC is locally defined, with a distribution over the entire interface. The other approach, the lumped one, is related to the general characteristics of the surfaces present in the interface. In this case, TCC is taken as a constant value over the entire interface, considering the average rate of heat flux to temperature drop in all micro regions along the interface.

Heat transfer in the interface between two materials is a complex process and it is affected by many factors [7], the main ones being:

(a) Pressure;
(b) Mean distance between the surfaces in the interface;
(c) Thermal conductivity of the materials (k);
(d) Surfaces properties: hardness and roughness;
(e) Material properties: yield strength and modulus of elasticity; and
(f) Average temperature in the interface.

This large number of parameters and the microscopic characteristics of the thermal process in the interface result in a great complexity to accurately determine the TCC between two materials. Therefore, most of the experimental techniques to evaluate TCC and its available data are focused on its average value. However, new applications, such as component miniaturization, biotechnology and nanotechnology, have improved the development of methods and techniques to estimate TCC in a more precise way as well as its distribution across the interface. Hence, some researches are focused in using more accurate and detailed characteristics of the contacting surfaces between materials. Following this line, mathematical and statistical models have been created to simulate micro irregularities on the interface between two materials by developing roughness patterns. Zhao *et al.* [8], predicted the TCC based on the contact peaks of irregular surfaces on two-dimensional statistics models of the roughness profile. To develop the model, they measured the roughness profiles of some common machined surfaces. The results showed that is very difficult to establish a complete topography of the surfaces and the authors concluded that the model still needs to be refined to a three-dimensional version in order to be more effective.

More recent works focused not only in the averaged TCC value, but in the determination of its spatial distribution on the whole interface as well as its variation in time. Yang [9] presented the solution of a transient inverse heat transfer problem to estimate the TCC in single-coated optical fibers. In the work of Shojaeefard *et al.* [10], a transient inverse heat transfer problem was used to estimate TCC in periodically contacting surfaces. The inverse