



Effect of swelling on the onset of buoyancy-driven convection during the CO₂ dissolution process in a porous medium



Min Chan Kim

Department of Chemical Engineering, Jeju National University, Jeju 690-756, Republic of Korea

ARTICLE INFO

Article history:

Received 10 October 2015

Received in revised form 25 April 2016

Accepted 26 April 2016

Available online 19 May 2016

Keywords:

Buoyancy-driven convection

Porous medium

Swelling effect

Linear stability analysis

Spectral analysis

ABSTRACT

By considering the swelling due to the CO₂ dissolution, the onset of buoyancy-driven convection in a porous medium is analyzed theoretically. Based on the Darcy equation, the flow and advection–diffusion equations are derived. The base concentration field in an interface movement system due to the volume expansion is slightly different from a fixed interface one. Based on the linear stability theory, new stability equations are obtained in the global (τ, z) -domain and in the semi-infinite (τ, ζ) -domain. Unlike the stability equations in the global (τ, z) -domain, a normal mode analysis is possible in the semi-infinite (τ, ζ) -domain. The normal mode stability equations are solved analytically for the no-swelling system, and numerically for the swelling ones. The critical conditions are obtained as a function of the Darcy–Rayleigh number and the interface movement parameter. The interface movement accelerates the onset of convection with small wavelength instability. The effect of interface movement on the stability is severe, especially in a highly soluble system such as a CO₂ improved oil recovery case.

© 2016 Elsevier Ltd. All rights reserved.

1. Introduction

In a geological CO₂ storage system, CO₂ is gradually dissolved into a brine saturated in the porous matrix. The dissolution of CO₂ increases the density and volume of the CO₂/brine mixture [1]. Therefore, the CO₂ dissolution into the brine through the upper boundary can induce the onset of buoyancy-driven convection. Since this buoyancy-driven convection can accelerate CO₂ dissolution into the brine, many researchers have analyzed the gravitational stability of the geological CO₂ storage system. Furthermore, in a CO₂ improved oil recovery (CO₂-IOR) system, CO₂ can be dissolved in the oil as rich as 70–80 CO₂-mol% [1]. This high CO₂ solubility can drastically decrease viscosity and can result in oil swelling by as much as 50–60 vol.% [1,2]. Swelling due to the CO₂ dissolution gives rise to an increase in oil recovery, and viscosity reduction improves mobility [3]. Also, CO₂ injection into an oil reservoir may reduce global warming through the CO₂ sequestration. Therefore, a better understanding of the CO₂ injection mechanism is critical in the design and operation of the CO₂-IOR process.

The analysis of gravitational instability in a fluid layer confined within a porous matrix begins with the Horton–Rogers–Lapwood (HRL) convection [4,5]. They examined thermally driven instability by adopting the methods developed for a homogeneous fluid. The extension of the classical HRL convection was well summarized by

Nield and Bejan [6]. In the above mentioned studies, it was assumed that there was a linear increase in temperature with depth, appropriate for gradual heating or for a steady state, e.g., the naturally occurring geothermal gradients in the subsurface. However, in the CO₂ sequestration and CO₂-IOR systems, there is a relatively rapid change in solute concentration at one boundary. The basic profile of concentration before the onset of convection is then time dependent and nonlinear. The related instability analysis has been conducted by using the initial value problem (IVP) approach [7,8]. Also, the stability of time dependent base states has been investigated by an energy method [7] and its modification [9]. A comparison among the different methods can be found in Emami-Meybodi et al.'s extensive review [10]. Also, non-linear analyses to predict the flow characteristics after the onset of convection have been actively conducted [7,11–13].

Recently, based on the non-modal analysis concept, Rapaka et al. [14,15] considered the maximum amplification over all possible infinitesimal perturbations introduced at the same time and compared their results with those from the linear amplification theory which has been used widely [8,16]. Later, Daniel et al. [12] determined optimal amplifications by solving the physical IVP and adjoint IVP simultaneously. However, all the above mentioned work has been done for the non-swelling mixtures. Very recently, Meulenbroek et al. [17] and Myint and Firoozabadi [18] studied the effect of swelling on the onset of a buoyancy-driven convection in a liquid saturated porous medium. Since the swelling

E-mail address: mckim@cheju.ac.kr

ratio of a CO₂/oil mixture is much larger than a CO₂/water system, the effect of the swelling on the gravitational instability is important in the CO₂-IOR system.

In the present study, taking the interface movement due to the swelling into account, the onset and growth of buoyancy-driven instability in the CO₂ dissolution process is analyzed theoretically. Under the linear stability analysis, the new stability equations are re-derived in the semi-infinite domain and solved without the QSSA. Through the initial growth rate analysis, the most dangerous mode of disturbance is identified. The critical conditions for the onset of convection are obtained as a function of the Darcy-Rayleigh number and the interface movement parameter.

2. Governing equations and base field

As shown in Fig. 1, a Newtonian fluid layer saturated in a porous medium is considered here. The porous medium is assumed to be homogeneous and isotropic, and its porosity and permeability are Π and K , respectively. Initially the fluid whose depth d is at rest, and there is no CO₂ dissolved in the fluid. It is assumed that CO₂ is continuously supplied from the top, the CO₂-liquid interface is sharp, and at the CO₂-liquid interface, the fluid is saturated with CO₂ with a fixed mass fraction C_{sat} . As CO₂ dissolves into the liquid layer, its volume increases due to the swelling and therefore, the interface position moves upward.

The flow and concentration fields to describe the present system are governed by

$$\nabla \cdot \mathbf{U} = 0, \quad (1)$$

$$\frac{\mu}{K} \mathbf{U} = -\nabla P + \mathbf{k} \rho_L \beta g C, \quad (2)$$

$$\Pi \frac{\partial C}{\partial t} + \mathbf{U} \cdot \nabla C = \Pi \mathcal{D} \nabla^2 C. \quad (3)$$

Here, we used the Darcy equation to model the flow through the porous medium and the Boussinesq approximation to explain the density change variation with respect to the CO₂ concentration. Since we are focused on the interface movement rather than the fluid compressibility, Eq. (1) is quite enough to explain the mass conservation. Also, \mathbf{U} is the Darcy velocity, μ is the viscosity, P is the pressure, ρ_L is the density of pure liquid, C is the CO₂ concentration, g is the gravitational acceleration, and \mathcal{D} is the effective diffusion coefficient. The unit vector in the direction of the gravitational acceleration is \mathbf{k} . To consider the interface movement effect, the variation of the interface position, H , should be considered. Recently, by considering the mass balance across the interface, Myint and Firoozabadi [18] suggested the following interface condition:

$$-\frac{dH}{dt} = \frac{\mathcal{D}}{1 - C_{sat}} \frac{\partial C}{\partial Z} \text{ at } Z = -H(t). \quad (4)$$

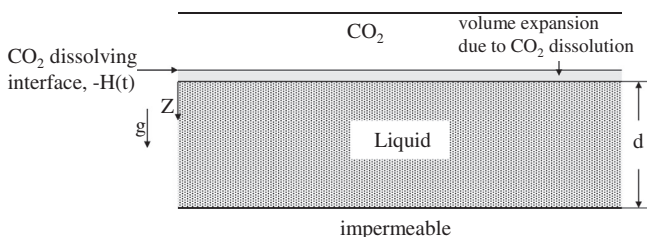


Fig. 1. Schematic diagram of system considered here. Volume expansion due to the CO₂ dissolution makes the interface move upwardly.

By using $L(= d/Ra_D)$, $\Pi \mathcal{D}/L$, $K/(\mu \Pi \mathcal{D})$, C_{sat} and $L^2/(\Pi \mathcal{D})$ as length, velocity, pressure, concentration, and time scaling factors, respectively, the above governing equations are given in the dimensionless form as

$$\nabla \cdot \mathbf{u} = 0, \quad (5)$$

$$\mathbf{u} = -\nabla p - \mathbf{k}c, \quad (6)$$

$$\frac{\partial c}{\partial \tau} + \mathbf{u} \cdot \nabla c = \nabla^2 c, \quad (7)$$

$$-\frac{dh}{d\tau} = \frac{C_{sat}}{1 - C_{sat}} \frac{\partial c}{\partial z} \text{ at } z = -h(\tau). \quad (8)$$

The proper boundary conditions are

$$\mathbf{u} = 0 \text{ and } c = 1 \text{ at } z = -h(\tau), \quad (9a)$$

$$\mathbf{u} = 0 \text{ and } \frac{\partial c}{\partial z} = 0 \text{ at } z = Ra_D. \quad (9b)$$

It should be noted that the interface position $h(\tau)$ is not determined yet. Here the important dimensionless group Ra_D is the Darcy-Rayleigh number defined as

$$Ra_D = \frac{g \beta C_{sat} K d}{\Pi \mathcal{D} \nu},$$

where ν is the kinematic viscosity of pure liquid.

Before the onset of convection, the basic diffusion state is represented as

$$\frac{\partial c_0}{\partial \tau} = \frac{\partial^2 c_0}{\partial z^2}, \quad (10)$$

under the following initial and boundary conditions:

$$c_0(0, z) = 0, \quad c_0(\tau, -h) = 1 \text{ and } (c_0(\tau, Ra_D) = 0. \quad (11)$$

For the limiting case of very small τ , the solution of the above one-dimensional diffusion equation can be written as [19]

$$c_0(\tau, z) = A + B \operatorname{erf}\left(\frac{z}{2\sqrt{\tau}}\right). \quad (12)$$

By applying the initial and boundary conditions (11) into Eq. (12), the base concentration can be obtained as

$$c_0 = \frac{\operatorname{erfc}(\zeta/2)}{\operatorname{erfc}(\eta/2)}. \quad (13)$$

If we combine Eqs. (8) and (13), we can obtain the interface position $h(\tau)$ as

$$h = \eta \sqrt{\tau} \quad (14)$$

and

$$\frac{\eta}{2} = \frac{C_{sat}}{1 - C_{sat}} \frac{1}{\operatorname{erfc}(\eta/2)} \frac{1}{\sqrt{\pi}} \exp\left(-\frac{\eta^2}{4}\right). \quad (15)$$

Myint and Firoozabadi [18] obtained the same results by considering the similarity between the Stefan problem and the present one. Meulenbroek et al. [17] pre-assumed Eq. (14) and determined the constant η from the experimental data. In Fig. 2, η is plotted as a function of C_{sat} .

Download English Version:

<https://daneshyari.com/en/article/656404>

Download Persian Version:

<https://daneshyari.com/article/656404>

[Daneshyari.com](https://daneshyari.com)