International Journal of Heat and Mass Transfer 100 (2016) 876–891

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Large scale three-dimensional topology optimisation of heat sinks cooled by natural convection

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ARTICLE INFO

Article history:
Received 18 August 2015
Received in revised form 2 May 2016
Accepted 4 May 2016

Keywords:
Topology optimisation
Heat sink design
Natural convection
Large scale
Multiphysics optimisation

ABSTRACT

This work presents the application of density-based topology optimisation to the design of three-dimensional heat sinks cooled by natural convection. The governing equations are the steady-state incompressible Navier–Stokes equations coupled to the thermal convection–diffusion equation through the Bousinessq approximation. The fully coupled non-linear multiphysics system is solved using stabilised trilinear equal-order finite elements in a parallel framework allowing for the optimisation of large scale problems with order of 20–330 million state degrees of freedom. The flow is assumed to be laminar and several optimised designs are presented for Grashof numbers between 10^3 and 10^6. Interestingly, it is observed that the number of branches in the optimised design increases with increasing Grashof numbers, which is opposite to two-dimensional topology optimised designs. Furthermore, the obtained topologies verify prior conclusions regarding fin length/thickness ratios and Biot numbers, but also indicate that carefully tailored and complex geometries may improve cooling behaviour considerably compared to simple heat fin geometries.

1. Introduction

Natural convection is the phenomena where density-gradients due to temperature differences cause a fluid to move. Natural convection is therefore a natural way to passively cool a hot object, such as electronic components, light-emitting diode (LED) lamps or materials in food processing.

Structural optimisation is the discipline of modifying the design of a structure in order to improve its performance with respect to some desirable behaviour. Structural optimisation techniques, such as size and configuration optimisation, are very efficient if a close-to-optimal design is already known, or the topology of the structure is dictated by e.g. certain manufacturing methods. These methods are frequently applied to the design of heat sinks in electronics cooling. The following literature review is by no means complete, giving only representative examples: Morrison [1] optimises plate fin heat sinks in natural convection using a downhill simplex method and empirical correlations. The design variables are the fin thickness, fin spacing and backplate thickness; Ladezma and Bejan [2] investigate the geometric arrangement of staggered vertical plates in natural convection using numerical simulations. The design variables are various dimensions of the staggered arrangement; Iyengar and Bar-Cohen [3] investigate vertical pin-fin, plate-fin and triangular-fin heat sinks in natural convection using analytical and empirical correlations. The design variables are the fin thickness and spacing; Bahadur and Bar-Cohen [4] optimise staggered pin-fin heat sinks for natural convection cooled microprocessor applications using analytical equations. The design variables are pin height, diameter and spacing; Jang et al. [5] optimise radial pin-fin heat sinks for LED applications using numerical simulation and a genetic algorithm. The design variables are the number and length of fins. Furthermore, there exists a vast literature treating optimal structures for surface-to-point and volume-to-point heat generation, e.g. [6], but these do not consider convective heat transfer and are thus not directly relevant to the problems at hand.

While parameter studies and simple optimisation techniques, like the abovementioned, can provide insight and improvements to existing designs, they are all limited in their design freedom as an *a priori* determined initial design must be given. Topology optimisation allows for a vastly expanded design space, allowing for the formation of unintuitive and unanticipated designs that fully exploit the governing physics. Topology optimisation is a material distribution method [7–9] used to optimise the layout of a structure. In order to take convective heat transfer, to an ambient fluid, into account in the design process of density-based methods, a common extension is to introduce some form of interpolation of the convection boundaries, see e.g. [10–13].

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2016.05.013
0017-9310/

More recently, these simplified models have been used by Dede et al. [14] to design and manufacture heat sinks subject to jet impingement cooling, as well as by Zhou et al. [15] in an industrial framework to optimise electric motor covers and heat sinks.

Generally, the application of a predetermined and design-independent convection coefficient is at best inaccurate and may have a strong influence on resulting designs and their performance. In practise, topology optimisation based on simplified models may lead to unanticipated designs and closed cavities, thereby violating the assumptions of the simplified model [16]. During the optimisation process, the design changes significantly and the interaction with the ambient fluid changes as well. Therefore, to ensure physically correct capturing of the aspects of convective heat transfer, the full conjugate heat transfer problem must be solved. Obviously, the employment of a full-blown fluid model increases computational time and complexity considerably. Hence, using the simplified convection approach, that provides for very fast solution times, may likely provide a good first estimate for an optimised topology or may be used for post-processing once topology and associated local convection coefficients have been found. Furthermore, it may be beneficial to use the simplified convection approach, when the flow is too complex to model in current optimisation settings (hundreds or thousands of function evaluations). Please see [13,15–18] for further discussions on the strengths and weaknesses of using the simplified convection approach.

Topology optimisation for fluid systems began with the treatment of Stokes flow in the seminal article by Borrvall and Petersson [19] and has since been applied to Navier–Stokes [20], as well as passive transport problems [21,22], reactive flows [23], transient flows [24–26], fluid–structure interaction [27,28], amongst many others. The extension of topology optimisation to turbulent fluid flow is very much in its infancy [29] and requires further research. Conjugate heat transfer problems were first treated in [30,31] and is very much an active field of research today [32–37]. However, almost all work is focused on forced convection, where the fluid flow is induced by a fan, pump or pressure-gradient. The authors have previously presented a density-based topology optimisation approach for two-dimensional natural convection problems [18]. Recently, Coffin and Maute presented a level-set method for steady-state and transient natural convection problems using the eXtended finite element method (X-FEM) [38]. Interested readers are referred to [18] for further references and a deeper introduction to topology optimisation in fluid dynamics and heat transfer.

Throughout this article, the flows are assumed to be steady and laminar. The fluid is assumed to be incompressible, but buoyancy effects are taken into account through the Boussinesq approximation, which introduces variations in the fluid density due to temperature gradients. The inclusion of a Brinkman friction term facilitates the topology optimisation of the fluid flow.

The scope of this article is primarily to present and provide basic verifications of a large scale, three-dimensional framework for topology optimisation of thermal heat sinks. The methodology builds on the two-dimensional framework presented in [18]. Thus, only a brief overview of the underlying finite element and topology optimisation formulation is given and the reader is referred to [18] for further information. The numerical extension to three dimensions is non-trivial and hence the present article includes new discussions on interpolation and continuation strategies, as well as on computational issues arising from solving the large-scale, nonlinear equation systems considered. In this first application of topology optimisation to natural convection problems in three dimensions, dimensionless parameters and fictitious properties are used. Nevertheless, interesting insight will be gained on the effect of the Grashof number in optimal design. Ongoing and future work is devoted to the treatment of physically realistic problems for LED lamp coolers [39] and other practical devices.

In recent years, an increasing body of work has been published on efficient large scale topology optimisation. These works cover the use of high-level scripting languages [40,41], multiscale/-resolution approaches [42,43] and parallel programming using the message parsing interface (MPI) and C/Fortran [44–47]. To facilitate the solution to truly large scale conjugate heat transfer problems, the implementation in this article is done using PETSc [48] and the framework for topology optimisation presented in [47].

The layout of the article is as follows: Section 2 presents the governing equations; Section 3 presents the topology optimisation problem; Section 4 briefly discusses the finite element formulation; Section 5 discusses the numerical implementation details; Section 6 presents scalability results for the parallel framework, optimised designs for two test problems, as well as verification results; Section 7 finishes with a discussion and conclusion.

2. Governing equations

The dimensionless form of the governing equations have been derived based on the Navier–Stokes and convection–diffusion equations under the assumption of constant fluid properties, incompressible, steady flow and neglecting viscous dissipation. Furthermore, the Boussinesq approximation has been introduced to take density-variations due to temperature-differences into account. A domain is decomposed into two subdomains, $\Omega = \Omega_f \cup \Omega_s$, where Ω_f is the fluid domain and Ω_s is the solid domain. In order to facilitate the topology optimisation of conjugate natural convective heat transfer between a solid and a surrounding fluid, the equations are posed in the unified domain, Ω, and the subdomain behaviour is achieved through the control of coefficients. The following dimensionless composite equations are the result.

$\forall \mathbf{x} \in \Omega$:

$$u_j \frac{\partial u_i}{\partial x_j} - Pr \frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \frac{\partial p}{\partial x_i} = -\alpha(\mathbf{x}) u_i - Gr Pr^2 e_i^g T \tag{1a}$$

$$\frac{\partial u_j}{\partial x_j} = 0 \tag{1b}$$

$$u_j \frac{\partial T}{\partial x_j} - \frac{\partial}{\partial x_j}\left(K(\mathbf{x}) \frac{\partial T}{\partial x_j}\right) = s(\mathbf{x}) \tag{1c}$$

where u_i is the velocity field, p is the pressure field, T is the temperature field, x_i denotes the spatial coordinates, e_i^g is the unit vector in the gravitational direction, $\alpha(\mathbf{x})$ is the spatially-varying effective impermeability, $K(\mathbf{x})$ is the spatially-varying effective thermal conductivity, $s(\mathbf{x})$ is the spatially-varying volumetric heat source term, Pr is the Prandtl number, and Gr is the Grashof number.

The effective thermal conductivity, $K(\mathbf{x})$, is defined as:

$$K(\mathbf{x}) = \begin{cases} 1 & \text{if } \mathbf{x} \in \Omega_f \\ \frac{1}{C_k} & \text{if } \mathbf{x} \in \Omega_s \end{cases} \tag{2}$$

where $C_k = \frac{k_f}{k_s}$ is the ratio between the fluid thermal conductivity, k_f, and the solid thermal conductivity, k_s. Theoretically, the effective impermeability, $\alpha(\mathbf{x})$, is defined as:

$$\alpha(\mathbf{x}) = \begin{cases} 0 & \text{if } \mathbf{x} \in \Omega_f \\ \infty & \text{if } \mathbf{x} \in \Omega_s \end{cases} \tag{3}$$

in order to ensure zero velocities inside the solid domain. However, numerically this requirement must be relaxed as will be described in Section 3. The volumetric heat source term is defined as being active within a predefined subdomain of the solid domain, $\omega \subset \Omega_s$:

$$s(\mathbf{x}) = \begin{cases} 0 & \text{if } \mathbf{x} \notin \omega \\ s_0 & \text{if } \mathbf{x} \in \omega \end{cases} \tag{4}$$