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Review

Constructal design for convection melting of a phase change body

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ABSTRACT

This paper documents a numerical study of time-dependent melting dominated by natural convection in a cylinder filled with a phase change material. In accord with constructal design, the search is for effective heat-flow architectures. The volume-constrained improvement of the designs for heat flow begins with assuming the simplest structure, where a single line serves as heat source. Next, the heat source is endowed with freedom to change its shape as it grows. The objective of the numerical simulations is to discover the geometric features that lead to the fastest melting process. The results show that the heat transfer rate density increases as the complexity and number of degrees of freedom of the structure are increased. Furthermore, the angles between heat invasion lines have a minor effect on the global performance compared to other degrees of freedom: number of branching levels, stem length, and branch lengths. The effect of natural convection in the melt zone is documented.

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1. Introduction

The evolution of flow designs is a widespread occurrence in engineering and nature. One design approach that has gained increasing attention in recent years is based on the constructal law [1], which is the thought that geometry (flow architecture) is generated in the pursuit of global performance subject to global constraints, in flow systems that are free to morph. The growing interest in the design approach is due to its implications in the field of heat transfer and fluid flow with the objective of achieving greater heat transfer rate density in a finite volume [2–24].

In this article, we apply the method of constructal design to latent heat storage. Energy storage is essential in designs for the effective utilization of unsteady and random renewable energy (e.g., solar energy and wind energy) by storing the energy in the form of latent heat during the high-availability period, for later use.

Traditionally, the study of latent heat storage is based on models where the heating entity, wall or embedded pipe, is solid and stationary. In such models, the melting of the phase-change material proceeds in the direction perpendicular to the stationary solid surface. In the present study we show that the model with stationary heating surface corresponds to sufficiently long times, when the flowing heating agent has had time to travel along and occupy

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Nomenclature

B	volume, m^3
c_P	specific heat at constant pressure, kJ/kg K
D	enclosure diameter (height), m
F	buoyancy force, N
g	gravity acceleration, m/s^2
k	thermal conductivity, W/m K
L_f	latent heat of fusion, J/kg
L_0	stem length, m
L_1	branch length, m
n	number of bifurcation levels
P	pressure, Pa
Ra	Rayleigh number
t	time, s
t_f	duration of melting by conduction, s
T	temperature, K
T_0	melting temperature, K
T_1	invading line temperature, K
u	velocity vector, m/s
V	speed of invading line, m/s

Greek symbols

α	tilt angle, degree
β	thermal expansion coefficient, K^{-1}
β_1	bifurcation angle, degree
δ	radial thickness of the melted domain, m
δ_f	thickness of the melted layer at the end of the conduction mode, m
ΔT	excess temperature
μ	dynamic viscosity, Pa s
ρ	density, kg/m^3
ν	kinematic viscosity
τ	temperature interval, K

Subscript

c	end of consolidation phase

Symbol

$\sim$	nondimensional

the entire extent of the solid surface. This long-times behavior will be defined as the "consolidation" period in the present model, where the study of the heating and melting process begins earlier, when the heating agent begins to invade the surface that immediately begins to heat and melt the surrounding phase-change material. This behavior at short time will be defined as the "invasion" phase.

We showed in an earlier study that this complete model, with invasion followed by consolidation, accounts for the S-shape of the growth of the melt volume over time [25]. The melting rate is initially small, it then increases to a maximum, and finally it decreases monotonically until the melting is complete. The objective of the present study is to determine the relationship between the configuration of the invasion path of the heating effect and the maximum melting rate, or the steepness of the S-shaped history of the volume of melt. We seek to determine three-dimensional heat invasion architecture that offers the shortest time for melting the entire amount of phase-change material.

2. Model

Consider the time-dependent melting of a phase-change material in a cylindrical enclosure heated by a growing line of higher temperature (T_1) placed on the axis, Fig. 1. The line tip speed is V. Initially, the volume $B = \pi D^3/4$ is filled entirely by the solid phase, the initial temperature T_0 of which is uniform and identical to the melting temperature. The material properties are: $k = 0.2$ W/m K, $\rho = 800$ kg/m^3, $c_P = 1.25$ kJ/kg K, $L_f = 125$ kJ/kg, $\mu = 0.008$ Pa s, $T_0 = 303$ K, $\tau = 1$ K, $\beta = 0.002$ K^{-1}, where τ is the temperature range over which melting occurs. The boundary of this volume is modeled as adiabatic. Beginning with the time $t = 0$, heat begins to flow from the heat source to the phase change material and as time passes, a finger-shaped melt volume continues to grow along the advancing line [26].

The Navier–Stokes and the heat transfer equations are coupled to model natural convection. The fluid is modeled as Boussinesq-incompressible, in other words, the variation in density ρ is negligible everywhere except in the body force term of the momentum equation

$$F = \rho g[-1 + \beta(T - T_0)] \tag{1}$$

where β is the coefficient of thermal expansion. With this assumption, the governing equations of conservation of mass, momentum and energy can be written as follows:

$$\nabla u = 0 \tag{2}$$

$$\rho \frac{Du}{Dt} = -\nabla P + \mu \nabla^2 u + F \tag{3}$$

$$\rho c_P \frac{DT}{Dt} = k \nabla^2 T \tag{4}$$

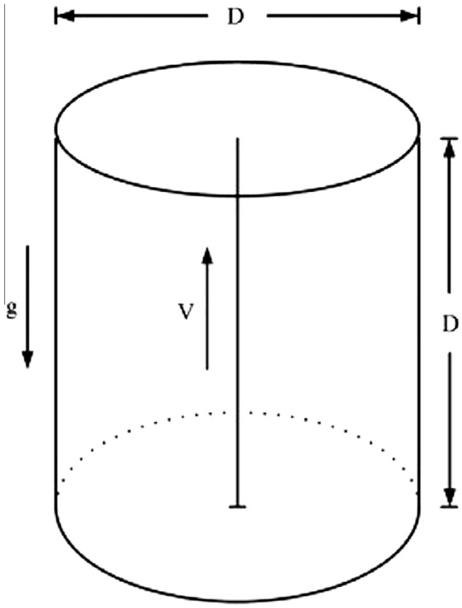

Fig. 1. Cylindrical enclosure filled with phase change material (PCM) heated by a single invading line.