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Extended lubrication theory for generalized Couette flow through converging gaps

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ABSTRACT

The lubrication theory based approximation of the Navier–Stokes equations represents a widely used approach for the computation of viscous force dominated flows through narrow channels or gaps. Its inherent neglect of advective transport becomes increasingly questionable when considering strongly converging geometries and particularly when considering heat transfer as well. The present work develops an extended lubrication theory based model, which accounts for inertia and convective heat transfer in terms of first-order perturbations. The extended model is applied to generalized Couette flow through strongly converging annular gaps typically met in wire coating dies, where the moving wall-driven motion is characterized by very high local shear rates and steep axial pressure gradients. The evaluation of the analytical solutions against numerical results from CFD simulations demonstrates the scope and the limits of the original lubrication theory approximation in describing the local flow and thermal conditions inside the gap. Including first-order convective contributions the extended model is shown to provide a significantly improved description, especially of the redistribution of the generated viscous heat. The most pronounced improvements are shown by the temperature profiles predicted at axial positions with rapidly changing cross-section for fluids with higher Prandtl numbers. The observed gain in accuracy is also reflected by very accurate predictions of the total transfer rates at the boundaries in the global balances of heat and momentum. Using the presently proposed extension the lubrication theory based model represents a reliable and computationally efficient approach for investigating the flow and heat transfer inside narrow gaps, which is well applicable to a wide range of different gap geometries.

1. Introduction

Viscous laminar flow through converging gaps has been widely studied due to its high technical relevance. Typical applications are found in the processing of polymer melts, liquid foods and drugs, or in coating technology, like the coating of magnet wires with a thin layer of insulation. The present work particularly investigates generalized Couette flow inside cylindrical gaps, which is essentially characterized by a shear-driven motion of the fluid induced by a moving wall, and high axial pressure gradients due to the axial variation of the gap height. As such, the presently investigated type of flow is well representative for a typical wire coating process in enameling dies, as schematically sketched in Fig. 1. The primary-coated wire being drawn through the die acts here as a moving inner wall driving the flow. Due to the converging geometry the abundant fresh enamel of the primary coating is removed, and the wire finally exits the die coated with a layer of defined thickness. The computational studies performed on such a flow configuration thus far mostly assumed the approximation of the lubrication theory, putting often much emphasis on the effect of non-Newtonian flow behavior. Assuming, e.g., a power-law fluid Flumerfelt et al. [1] presented a lubrication theory based analytical solution for planar gaps, while Lin and Hsu [2] analyzed axisymmetric flow through concentric annuli. Later, Lin [3] included heat transfer into the computations. A comprehensive review of the basic concepts for the computational analysis of Couette-type flow and heat transfer in converging gaps was presented by Mitsoulis [4]. Carley et al. [5] computed the flow and temperature field inside the conical section of wire coating dies solving numerically a simplified set of the two-dimensional conservation equations with a semi-implicit finite-difference scheme. Dijksman and Savenije [6] presented a lubrication theory based analytical solution for a converging annular gap flow introducing a special toroidal coordinate system. Shah et al. [7] carried out an extensive analytical computation of both flow and heat transfer of power-law fluids inside cylindrical gaps with constant cross-section.

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Nomenclature

c_p	specific heat capacity [J/kg K]
F_D	drag force [N]
H	enthalpy flow rate [W]
h	die exit gap height [m]
L	axial length of the die [m]
n_{geo}	geometry parameter [–]
P_η	production due to the viscous heating [W]
p	pressure [Pa]
Q_w	heat flow into the wire [W]
r	radial coordinate [m]
r_w	radius of the wire [m]
r_d	contour of the die [m]
T	thermodynamic temperature [K]
U_w	velocity of the wire [m/s]
u	axial velocity component [m/s]
$\dot{V}$	volumetric flow rate [m^3/s]
v	radial velocity component [m/s]
z	axial coordinate [m]

Greek symbols

δ	difference in gap height between die inlet and outlet [m]
ε	expansion parameter for velocity [–]
ε_T	expansion parameter for temperature [–]
η	dynamic viscosity [Pa s]
λ	thermal conductivity of the fluid [W/m K]
ρ	density of the fluid [kg/m^3]

Dimensionless numbers

Ec	Eckert number [–]
Nu	Nusselt number [–]
Pe	Péclet number [–]
Pr	Prandtl number [–]
Re_L	Reynolds number [–]

Superscripts

*	dimensionless quantity

Subscripts

0	zeroth-order solution
1	first-order solution
d	on the inner surface of the die
w	on the surface of the wire

The neglect of the inertia as represented by the advective transport terms becomes increasingly questionable in the case of strongly converging cross-sections. This simplification markedly limits the scope of the lubrication theory approximation, when applied to real-file die geometries. Appropriate extensions to the lubrication theory have thus far been proposed only for other narrow gap flow configurations, where the liquid is differently set in motion and/or the gap geometries are different. Extended analytical solutions were obtained by Collins et al. [8] for flow in planar journal bearings, and by Tavakol et al. [9] for pressure-driven flow through narrow planar channels with axially varying channel height. The latter non-Couette type flow is of special relevance in biomedical applications considering, e.g., human blood flow through arteries with varying cross-section due to stenosis as investigated by Nadeem and Ijaz [10], or peristaltic liquid motion driven by a periodic wave-like contraction of the channel walls [11]. These very recent computational studies on biomedical flow commonly assume only mild contractions in cross-section, so that they can neglect inertia and convective heat transfer to obtain still fairly accurate exact solutions of the problem. Hall et al. [12] computed analytical solutions for pressure-driven converging flow between stationary coaxial cones, where they considered very low Reynolds number (Stokes flow) conditions as well as high Reynolds number flow with non-negligible inertia. A special Couette-type flow evolving inside a narrow gap between a solid disc and a plate, assuming one wall rotating and the other as stationary, was analyzed by Shevchuk [13], who computed a self-similar solution of the full set the two-dimensional conservation equations of mass, momentum and heat. This concept represents a very powerful analytical approach, in that it basically imposes no restrictions on Reynolds or Prandtl numbers as typically met in the lubrication theory approximation. The actually considered flow configuration and boundary conditions must permit a self-similarity transform of all dependent variables though.

The present work attempts to improve the predictions of the lubrication theory approximation for generalized Couette flow by including the effect of the advective transport in terms of corresponding first-order perturbations. The perturbed solutions shall particularly provide more accuracy when considering strongly converging geometries associated with a rapid axial decrease in gap height. Accounting for the convective transport of heat shall also improve the predictions for the temperature field, especially for Prandtl numbers higher than unity. The possible improvement of the predictions of the extended lubrication theory

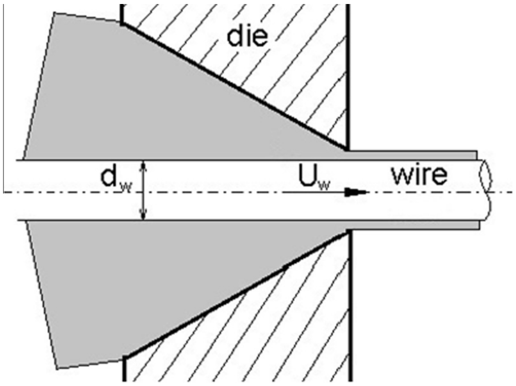

Fig. 1. Wire coating die principle.

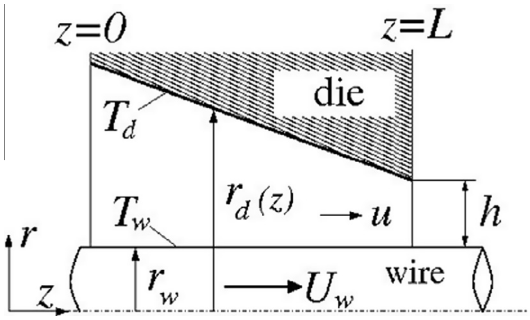

Fig. 2. Converging gap domain.