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Effect of entropy waves on transient energy growth of flow disturbances in triggering thermoacoustic instability

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ABSTRACT

In a linearly stable but nonlinearly unstable thermoacoustic system, thermoacoustic instability may be triggered by small-amplitude acoustic perturbation undergoing transient growth. The present work study the effect of entropy waves on transient energy growth of acoustic disturbances in triggering thermoacoustic instability. For this, modal and non-normality analysis of these thermoacoustic systems is performed first. It is shown that the systems are non-normal due to the non-orthogonality of the entropy modes and the acoustic modes. The contribution to the non-orthogonality of the acoustic modes is identified to consist of (1) the non-trivial boundary conditions, (2) unsteady heat release and (3) the mean flow present. To quantify the amplification of the energy of the acoustic disturbances, transient growth is characterized and calculated. It is found that the initial acoustic energy can be amplified by a factor of the order of 10^3, if the system is ended with choked outlet. It is 2-order greater than the G_{ac}^{max} (10^0–10^1) observed in the open-ended thermoacoustic system. Such dramatic amplification is mainly due to the energy transfer occurred at the system outlet from entropy to acoustic modes. The detailed analysis reveals that the entropy waves in the choked system have much greater potential to trigger thermoacoustic instability than expected. Neglecting the entropy waves might lead to wrong prediction of the transient growth of acoustic disturbances in triggering thermoacoustic instability.

1. Introduction

Thermoacoustic instability characterized by large-amplitude pressure oscillations occurs in many combustion systems [1–5] or in thermoacoustic engines [6–8]. It arises from the coupling between the unsteady heat release and flow disturbances [9–12]. Linear stability analysis is generally conducted as a conventional tool for studying the stability behaviors of a thermoacoustic system. A thermoacoustic system is linearly stable, when all eigenmodes decay exponentially. However, even when all the eigenmodes are stable, sometimes large amplitude oscillations can also occur. This is triggering [13], which can be observed in a system 'linearly stable but nonlinearly unstable'. By making use of 'flame describing function (FDF)', the nonlinear combustion oscillations are analyzed by Noiray et al. [14]. They showed that the key element of the triggering is the dependence of the phase of FDF on driving velocity amplitude. In practice the triggering can be caused by a large-amplitude impulse such as a bomb in a combustion chamber. And it can also be evolved from a small perturbation, which is of the order of background noise. For the latter case, transient growth is responsible for the stage from the initiation of a small perturbation to the threshold of triggering. The transient growth and triggering of nonlinear instability in a lean-premixed gas turbine combustor were measured and analyzed by Kim and Hochgreb [15] by using linear and nonlinear flame transfer function methodologies. Transient growth is important in triggering nonlinear oscillations and people recently have found that non-normality may play an important role in transient growth.

In the problem of hydrodynamic instability it is well known that due to the non-normality of the linear stability operator, small perturbations can grow transiently even when the system is linearly stable. Thus turbulence is triggered. Such mechanisms are revealed by Trefethen et al. [16], Butler and Farrell [17] and Reddy and Henningson [18]. The recent developments in the hydrodynamic field were reviewed by Schmid [19]. The non-normality and transient growth of a thermoacoustic system is first investigated by Balasubramanian and Sujith [20,21], and receive more attention recently. Generally the linearized governing equations of a thermoacoustic system are non-normal and the eigenvectors

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Nomenclature

A_p	the combustor cross-sectional area, m^2
c	sound speed, m/s
C_p	specific heat at constant pressure, kJ/kg K
E_{ac}, E_s	acoustical and entropy energy, J/m^2
E_{tot}	total energy, J/m^2
E_{cas}	energy containing cross terms of acoustics and entropy, J/m^2
F_{tot}	Cholesky decomposition component
G_{tot}	transient energy growth of total energy
G_{ac}	transient energy growth of acoustical energy
G^{max}	maximum transient energy growth
H	conjugate transpose
I_p, I_u, I_s	acoustic potential, kinetic and entropy energy intensity, W/m^2
K_t	the ratio of stagnation temperature across the heat source
$\boldsymbol{k}$	vector containing the coefficient k_j
L	length of the combustor inlet to nozzle inlet m
L_{eff}	the effective length of the nozzle, m
M_1, M_2	Mach number in pre- and after-heating regions
M_{tot}	a positive Hermitian matrix
$\aleph$	interaction index
N	number of eigenmodes
p	instantaneous pressure, Pa
P_{01}, P_{02}	stagnation pressure in pre and after-heating regions, Pa
q	complex flow disturbances
Q'	unsteady heat release rate, J/m^3
Q	instantaneous heat release rate, J/m^3
R_c	strength reflected acoustic waves by entropy disturbances at outlet boundary
R_{su}	parameter characterizing the efficiency of the entropy wave generation
s	entropy wave, J/kg K
t	time, s
τ	time delay, s
T_0	period of fundamental acoustic mode, s
T_{01}, T_{02}	stagnation temperature in pre- and after-heating regions, K
ΔT	mean temperature difference across the heat source, K
u	instantaneous velocity, m/s
$\mathbf{V}$	a complex vector
W	a weight matrix
$\mathbf{X}$	coefficient matrix
x_{f}	the axial heat source location, m
x_t	the total length of combustor and the nozzle, m
α	wave number
γ	ratio of specific heats
ω	eigenfrequency, rad/s
ω_i, ω_j	real and imaginary part of eigenfrequency ω, rad/s
$\mathbf{\Omega}$	diagonal matrix
ρ	instantaneous density, kg/m^3
$< >$	inner product of two eigenmodes

Overhat

–	mean value
^	frequency domain

Superscripts

$'$	fluctuating part
+	incident traveling wave
–	reflected traveling wave
*	the conjugate of the complex number

Subscripts

a	acoustics
r	real part
i	imaginary part
j	jth mode number

are non-orthogonal [22–24]. Even if all the eigenmodes decay, the flow disturbances may undergo transient growth. The transient growth of flow disturbances due to non-normality cannot be predicted by the modal analysis solution, which provides information only about the long-term evolution of the thermoacoustic system.

Juniper [25] investigated the transient growth and the triggering of nonlinear instability in a horizontal Rijke tube through linear and nonlinear procedures. The analysis is analogous to the bypass transition to turbulence in hydrodynamic instabilities. He showed that the initial perturbation can grow transiently towards an unstable periodic solution, and then grew to self-sustained oscillations (stable periodic solution). Such transient growth was due to the non-normality of the governing equations. He also calculated the most dangerous initial states. It was found that the 'safe operating region' is much smaller than linearized-predicted results, but only slightly smaller than nonlinear normal analysis. As shown in Fig. 13 [25], the energy of most dangerous initial states is about 12.5% lower than that of the unstable periodic solution, which also indicates that the non-normality is not significant.

In a thermoacoustic system, it has been shown that the non-normality are caused by unsteady heat release [20,21] and non-trivial boundary conditions [26]. In the previous studies we can find that the transient growth of the acoustical energy caused by non-normality can amplify the initial energy by a factor, $G_{ac}^{\max}$, of the order of 10^0 to 10^1 [25,27]. However in the problem of hydrodynamic instabilities the transient growth can amplify the initial energy by a factor of the order of 10^3 [17,18]. And this leads to the triggering of nonlinear instability by small perturbations of the order of back ground noise. Therefore we can see that the value of $G_{ac}^{\max}$ in a thermoacoustic system caused by the non-normality of acoustic mode is far lower than that in hydrodynamic instability problems.

The problems, however, become complicated when the flow effects are considered. The entropy and vorticity disturbances can be generated and convected with the mean flow. Wieczorek et al. [26] argued that the total energy of acoustic and entropy disturbances should be used to characterize the transient growth when mean flow effect is considered. They calculated the transient growth of total energy of the thermoacoustic system based on the Myers' energy norm [28]. The maximum transient growth can amplify initial total disturbance energy as high as 18 times. But with a choked outlet, the acoustical energy transient growth can amplify acoustical energy 6000 times [26], which is far different from the value calculated by using total energy. They argued that the non-normality should be measured based on total energy. As has been discussed, however, the non-linear instability can be triggered if the acoustic disturbance reaches the threshold in a system 'linearly stable but nonlinearly