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# Estimations and bounds of the effective conductivity of composites with anisotropic inclusions and general imperfect interfaces

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ABSTRACT

The present work aims to determine the effective thermal conductivity of composites made of an isotropic matrix phase in which circular or spherical inhomogeneities are embedded. The inhomogeneity phases can be anisotropic and the interface between the inhomogeneity and matrix phase can be modeled by a general thermal imperfect interface model across which both the temperature and normal heat flux across can suffer a discontinuity. To achieve this objective, we derive first a unified and exact solution for the thermal fields of the inhomogeneity problem consisting of a spherical or circular anisotropic inhomogeneity inserted via a general thermal imperfect interface into an infinity isotropic matrix medium subjected to a remote uniform loading at its external surface. Unlike the relevant results in elasticity, the intensity and heat flux fields inside circular and spherical inhomogeneities are shown to remain uniform even in the presence of the general thermal imperfect interface and anisotropy of inhomogeneity. Next, with the help of the foregoing solution results for the heterogeneity problem, the differential scheme is extended to predicting the effective thermal conductivity of composites with taking into account the imperfect interfaces between the constituent phases. Finally, the minimum potential and complementary energy principles and the morphologically representative pattern approach based on the Hashin–Shtrikman variational principles and the variational polarization principles are applied to such inhomogeneous materials and to bracketing their effective thermal properties. By constructing trial appropriate intensity and heat flux fields, the first- and second-order upper and lower bounds are obtained for the effective thermal conductivity of multiphase materials consisting of spherical or circular inhomogeneities embedded in a matrix. The estimations obtained by the differential scheme for the effective conductivity are shown to comply with the first- and second-order upper and lower bounds. Numerical results are provided to illustrate the dependence of the effective conductivity on the sizes of inhomogeneities and to compare the estimations with the relevant upper and lower bounds.

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## 1. Introduction

In the mechanics and physics of composite materials, most of the classical approximation schemes dedicated to estimating the effective properties of composite materials adopt often the assumption that the interfaces between the constituent phases are perfectly bonded. In the context of thermal conduction, an interface is called perfectly bonded if and only if across this interface, both temperature and normal heat flux are continuous. However, in many situations of practice, for example with presence of roughness or mismatch between the phases, the assumption of perfect bonding is inappropriate. Therefore, the consideration of imperfect interfaces between the constituent phases of composites is unavoidable. In the context of thermal conduction phenomenon, among all the linearly thermal imperfect interface models, the most widely used thermal imperfect interface models are the highly conducting (HC) interface model and the Kapitza's interface thermal resistance model, namely also lowly conducting (LC) interface model. First, the LC interface model stipulates that the normal component of the heat flux vector is continuous across an interface while the temperature across the interface suffers a jump proportional to the normal heat flux component (see e.g. [4]). The effect of thermal resistance interfaces on the effective conductivity of composites has been widely investigated (see e.g., [2,5–15]). Second, viewed as dual to the LC interface model, the HC interface one assumes that the temperature field is continuous across an interface while the normal heat flux is discontinuous across it (see e.g., [12,16–21,14]).

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Recently, the general thermal imperfect model has been proposed in the works of Gu et al. [22,23]. Unlike the two previous HC and LC imperfect interface models, the general thermal imperfect model, adopts a discontinuity of both the temperature and of the normal heat flux. The jumps the temperature and of the normal heat flux across the general imperfect interface must verify the some jump relations (see e.g. [22,23]). This imperfect interface model was at the beginning proposed on the basis of some phenomenological considerations and next derived rigorously with the aid of an asymptotic approach. More precisely, by considering a material surface or interface as the limit case of a very thin interphase situated between two bulk phases, the asymptotic approach showed that the Kapitza's interface thermal resistance model or the HC interface model can be derived from the general thermal imperfect model when the interphase is much less or more conducting than each of the constituents. In other words, the general imperfect interface model includes the two previous highly and lowly conducting imperfect interface models as particular cases. This asymptotic approach, closely related to some mathematical techniques of homogenization, was initiated with the works [1–3,5,18,24–26] in the general situation.

The present work is concerned with the determination of the effective thermal conductivity of multiphase composites with general thermal imperfect interfaces. All results obtained and elaborated methods in this work in the context of thermal conduction phenomenon are transposable to other transport phenomena like electric conduction, dielectrics, magnetism, diffusion and flow in porous media and to anti-plane elasticity, owing to their mathematical analogy and by means of appropriate physical interpretations. The composites studied in this work consist of a matrix in which anisotropic circular inclusions or spherical inclusions of different sizes are, in the two-dimensional (2D) or three-dimensional (3D) case, embedded via interfaces described by the general thermal imperfect interface model. The objective of the present work is threefold:

- First, it aims at obtaining a unified and exact solution for the thermal fields of the inhomogeneity problem in which a spherical or circular anisotropic inhomogeneity is embedded into an host infinity isotropic matrix medium subjected to a remote uniform loading at its external surface via a general thermal imperfect interface. By using an integral formulation, apart from the uniform part of the solution, the thermal disturbance of the solution due to the presence of the inhomogeneity and of the general thermal imperfect interface is decomposed into three parts which correspond to three sub-problems related to a prescribed uniform eigen-temperature gradient over the inhomogeneity, a temperature jump and a normal heat flux component jump at the imperfect interface. Each sub-problem can be exactly and analytically solved. Unlike the relevant results in elasticity, the intensity and heat flux fields inside circular and spherical inhomogeneities are shown to remain uniform even in the presence of the general thermal imperfect interface. Dissimilar to the results obtained in recent works with general thermal imperfect interfaces (e.g. [22,23]), the results derived, in the present work, for thermal fields of the inhomogeneity problem hold for any thermal anisotropy of the inhomogeneity phase. The heterogeneity problem under consideration in the present work plays an important role in micromechanics of linear composites. The solution obtained for this heterogeneity problem is needed to apply most of the well-known micromechanical schemes dedicated to estimating the effective properties of inhomogeneous materials, for example, the dilute, Mori–Tanaka, self-consistent, generalized self-consistent models to estimate the effective properties of heterogeneous materials.
- Second, it consists in extending the classical minimum potential and complementary energy principles and the morphologically representative pattern approach [27–29] based on the Hashin–Shtrikman variational principles [30] and the variational polarization principles [31] in elasticity to the thermal conduction phenomenon with taking into account the discontinuities and effects of interfaces between the matrix and inhomogeneity phases. By applying these variational principles and by constructing piecewise non-uniform trial intensity and heat flux fields, the first- and second-order upper and lower bounds can be derived for the effective thermal conductivity for such multiphase materials.
- Finally, it has the purpose of using the solution results obtained in the foregoing inhomogeneity problem and the extended differential scheme to estimate the effective thermal conductivity of multiphase composites. In contrary to results provided by applying the dilute, self-consistent schemes, we show that, in the presence of imperfect interfaces, the effective conductivity obtained by the differential approximation are always comprised between the generalized first- and second-order upper and lower bounds that are derived previously.

The paper is structured as follows. In Section 2, the phase constitutive laws of composites under investigation, the general thermal imperfect interface model and the general form of the effective thermal conduction behavior are specified. Section 3 is dedicated to deriving the solution for the thermal fields of the inhomogeneity problem consisting of a spherical or circular anisotropic inhomogeneity embedded into an host infinity isotropic matrix medium subjected to a remote uniform loading at its external surface via a general thermal imperfect interface. In Section 4, closed-form expression is obtained for the effective conductivity by using the differential scheme. In Section 5, by applying the classical minimum potential and complementary energy principles and by using the morphologically representative pattern approach, the first- and second-order upper and lower bounds are derived for the effective thermal conductivity of composites. In Section 6, the effects of general thermal imperfect interfaces and inhomogeneity sizes on the effective conductivity of composites are numerically discussed and illustrated. In Section 7, a few concluding remarks are provided.

## 2. Setting of the problem

We consider in this work a three-dimensional (3D) or two-dimensional (2D) composite (or multiphase material) consisting typically of $N\ (\geqslant 1)$ kinds of spherical or circular inhomogeneities embedded in a matrix. Each kind of inhomogeneities contains several identical spherical or circular inclusions which are assumed to be randomly distributed and orientated in the matrix phase. In other words, two spherical or circular inclusions belong to the same kind if and only if one may be obtained from the other one via a translation or/and rotation. The matrix and each inhomogeneity are assumed to be individually homogeneous. Relative to a 2D or 3D Cartesian coordinate system $\{x_1,\ldots,x_d\}$, with $d=2$ or 3, in a right-handed orthonormal basis $\{\mathbf{f}_1,\ldots,\mathbf{f}_d\}$, the matrix, referred to as phase 0, and the $i$th inhomogeneity, called phase $i$ with $i\in[1,N]$, have the linear thermal conduction behavior described by an anisotropic Fourier's law

$$\mathbf{q}^{(m)}=\mathbf{K}^{(m)}\cdot\mathbf{e}^{(m)}\quad\text{or}\quad\mathbf{e}^{(m)}=\mathbf{H}^{(m)}\cdot\mathbf{q}^{(m)}\tag{1}$$

where $\mathbf{K}^{(m)}$ and $\mathbf{H}^{(m)}=(\mathbf{K}^{m})^{-1}$, with $m=0,1,\ldots,N$, stand for the second-order tensors of thermal conductivity and resistivity of phase $m$, which are symmetric, positive definite and in general orthotropic. However, the present work is limited to the case where the matrix phase is assumed to be isotropic, so that the expressions for the

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