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# Optimize heat conduction problem using level set method with a weighting based velocity constructing scheme

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ABSTRACT

This paper is devoted to develop an efficient computational procedure for the level set-based topological design of heat conducting fields. Firstly, the level set model with a distance-suppression scheme (generalized Hamilton–Jacobi equation) is used to implicitly represent boundary of heat conductive material so that the periodical re-initialization can be avoided. Secondly, after demonstrating that the finite element thermal analysis takes the major portion of the total computational time, we present a weighting based velocity constructing method inspired from the conjugate gradient method to avoid performing finite element thermal analysis for solving the generalized Hamilton–Jacobi equation. Thirdly, a velocity renewing procedure and criteria for stopping the weighting method are developed for insuring the stability and a quick convergence. Finally, two dimensional topology optimization results of heat conduction problem under both single and multiple load cases are presented to demonstrate the validity of the proposed method.



## 1. Introduction

To design a mechanical device, the initial channel configuration setting is crucially important to the realization with high performance. For this reason, topology optimization has been gradually used at the concept level of the design process to arrive at a conceptual proposal, such that design development time and overall cost can be reduced while performance can be improved [1].

Topology optimization is regarded as an approach that optimizes material layout within a given design domain such that the required performance can be maximized [2]. With decades past, several numerical methods for topology optimization have been developed and they can be roughly classified into two categories. The first one is density-based methods, such as the homogenization method [3,4], the ESO (evolutionary structural optimization) method [5,6] and the SIMP (solid isotropic material with penalization) method [7,8]. The other one is the moving boundary methods, such as the level set method [9–11]. When compare with density-based methods, level set method does not involve mesh-dependent and checkerboard patterns. Furthermore, the level set method allows a clear description of the boundaries and thus can avoid ambiguities of intermediate material phases which are inevitable when using a density-based approach. Until now, these methods have been applied for solving a variety of engineering problems, such as compliant mechanisms design problem [12] and vibration problem [2]. For more information, readers are referred to several review articles, e.g., [13,14].

Thermal problems, such as design of radiative enclosures [15] and heat sink devices [16], have also been discussed in the context of topology optimization. Early efforts have been made by using the density-based methods [17,18] due to their relatively conceptual simplicity and easy of use. For example, Li et al. [19] used ESO method to solve the topology design problems subjected to steady heat conduction. Gersborg-Hansen et al. [20] used SIMP method to solve the heat conduction problem by using the finite volume method. Evgrafov et al. [21] considered the problem of optimal design of nano-scale heat conducting systems by developing a more accurate model by kinetic theory.

Recently, level set methods have been continuously applied on topology optimization of thermal problems. The earliest attempt can be found in [22]. After that, Zhuang et al. [23] discussed the heat conduction problem under multiple load cases. Yamada et al. [24] proposed a level set-based methodology based on the concept of the phase field method to solve thermal diffusive

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problems. Dealing with coupled thermal-fluid problem [25] and thermal conductors considering design-dependent effects [26] also have been conducted by the same research group.

Although level set methods have been widely utilized for solving topological design problems, the computational efficiency has received relatively less attention in spite of its significance. In fact, the conventional level set method for topology optimization concerned with obtaining optimal results by solving the Hamilton–Jacobi equation [9,10]. Meanwhile, the velocity for driving the evolution of the level set function is often set using the steepest descent method. It has been well recognized that, for solving the Hamilton–Jacobi equation, the time step has to satisfy the Courant–Friedrichs–Lewy (CFL) condition [27,28]. Usually the time step issued from this CFL condition is much smaller than the one which plays the role of the descent step in the minimization of the objective function. Furthermore, most of the current developed topology optimization methods include relatively time-consuming finite element analysis for the structure in each iteration step. As a result, bad efficiency is often indispensable to obtain an accurate optimal solution even if one performs several explicit time steps for solving the level set equation [22,29–31,25].

Removing periodical re-initialization can improve the efficiency, as mentioned in [32,33], where energy functionals are built into the level set equation. However, this only has a very small direct impact on efficiency because re-initialization only occupies a small fraction of the overall time. Most efforts are made by developing new schemes to set the velocity. For instance, a mapping algorithm is developed in [34] to speed up the evolving process of level set equation. Luo et al. [35] developed a semi-implicit level set method for structural topology optimization which allows enhanced relaxation on the time-step length. The efficiency can be greatly improved (for some cases, only 60 iterations are needed). Lately, a semi-Lagrangian level set method incorporated with a sensitivity modulation scheme is proposed in [36]. For this method, a line search algorithm is developed. A velocity predictor–corrector scheme is developed in [37] for solving structural topology optimization where an inner loop is inserted to update level set function separately. Both aforementioned methods can yield fewer design iterations and thus it improves the overall computational efficiency. Indeed, reducing the computational effort is a challenging topic for further development of topology optimization. Despite all the above attempts, efficient schemes for the level set based topology optimization are still needed.

In this study, we attempt to propose an efficient scheme to solve heat conduction problem. Firstly we use the level set model with a distance-suppression scheme to avoid the periodical re-initialization. After that we present a velocity constructing method by using weighting scheme inspired from the conjugate gradient method. This servers two main purposes. One is that the generated velocity can provide an alternative direction to decrease the objective function when comparing to the steepest descent direction. The other is that one can use the method for several extra iterations without performing the time-consuming elastic analysis. Two dimensional topology optimization results of heat conduction problem under both single and multiple load cases are presented as examples to demonstrate the validity of the proposed method.

The remainder of the paper is organized as follows. In Section 2, the basic concepts of the level set method are introduced and the generalized Hamilton–Jacobi equation for avoiding re-initialization is introduced. In Section 3, the design problem that is considered in this paper is introduced. In Section 4, a weighted equation which is inspired from the conjugate gradient method is developed to construct the velocity function. In Section 5, methods for ensuring the stability and a fast convergence of the optimization process are developed. The optimization algorithm is proposed. In Section 6, several numerical examples are presented to demonstrate the effectiveness of the proposed method. Finally, conclusions and a discussion for further work are put forward.

## 2. Level set method with a distance suppression scheme

We firstly describe the underlying idea of the level set method for topology optimization. Suppose that $D$ is the reference domain to contain all permissible shapes of the design domain $\Omega$ which consists of material sub-domain and voids. Boundary $\partial\Omega$ is the interface of the structure, and domain $D \setminus \Omega$ represents the void area. In level set method, the structural boundary $\partial\Omega$ is implicit embedded in a scalar function of a higher dimension $\phi$ [10,9] as its zero level set. Therefore

$$\begin{cases} \phi(\mathbf{x},t) > 0 & \text{if} \quad \mathbf{x} \in \Omega \\ \phi(\mathbf{x},t) = 0 & \text{if} \quad \mathbf{x} \in \partial\Omega \\ \phi(\mathbf{x},t) < 0 & \text{if} \quad \mathbf{x} \in D \setminus \Omega \end{cases} \tag{1}$$

where $t$ is the time, $\mathbf{x}$ is a point in the design domain.

The optimization process can be transferred into the evolution of the level set equation

$$\frac{\partial \phi}{\partial t} = V_n |\nabla \phi| \tag{2}$$

where $V_n$ determines the motion of the interface.

Generally, to ensure the stability of the optimization process, the gradient of the level set function near the zero level set needs to be controlled in order to avoid the level set function becoming too flat or steep. A so-called re-initialization procedure [27] is often used to maintain the level set function as a signed distance function, i.e., $|\nabla\phi| = 1$. Unfortunately, this will bring some unsolved issues, such as when and how to perform the re-initialization. Furthermore, such a numerical implementation is also time-consuming since additional partial differential equations (PDEs) need to be solved [34].

To overcome these issues, a distance suppression scheme has been developed in [33]. The underlying idea is to introduce an extra functional into the optimization model, such that a generalized Hamilton–Jacobi equation is actually used to update the level set function

$$\frac{\partial \phi}{\partial t} = V_n + \omega \nabla \cdot (d_r(|\nabla\phi|)\nabla\phi) \tag{3}$$

where $\omega$ is the weighting factor of the diffusion $\nabla \cdot (d_r(|\nabla\phi|)\nabla\phi)$ and set to

$$\omega = \frac{\Delta t_{\Re}}{\Delta t} \tag{4}$$

where $\Delta t_{\Re} = 0.5\Delta x / \max(|V_{\Re}|)$ and $\Delta t = 0.5\Delta x / \max(|V_n|)$ [33].

The diffusive rate $d_r(|\nabla\phi|)$ is defined as

$$d_r(|\nabla\phi|) = \frac{1}{|\nabla\phi|} \frac{\partial r(|\nabla\phi|)}{\partial |\nabla\phi|} \tag{5}$$

where $r(|\nabla\phi|)$ is the energy density function, which can be defined using

$$r(|\nabla\phi|) = \frac{1}{2}|\nabla\phi|^2(|\nabla\phi| - 1)^2 \tag{6}$$

Updating level set function using Eq. (3) will simultaneously minimize $r(|\nabla\phi|)$, which makes $|\nabla\phi|$ either be 0 or 1. This means the diffusion can maintain the level set function to close to a signed distance function near the zero level set, meanwhile forcing the gradient of the level set function to be 0 at locations far away from the zero level set. Not only can this avoid the periodically re-initialization, but also it can simplify the initialization process of the level set function. Merging the above equations we have