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# Numerical study of laminar mixed convection in a square open cavity

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ABSTRACT

The Lattice Boltzmann Method (LBM) is used to study steady and unsteady laminar flow in a channel with an open square cavity and a heated bottom wall in two dimensions, under mixed convection flow conditions. LBM is compared to results obtained by ANSYS-FLUENT for validation. Temperature, velocity and Nusselt number agree very well in the range of Reynolds and Richardson numbers studied, i.e. $50 \leqslant Re \leqslant 1000$ and $0.01 \leqslant Ri \leqslant 10$. Our observations indicate that the effect of the buoyancy force is negligible for $Ri \leqslant 0.1$, for all values of the Reynolds number considered. For Ri = 1, 10 buoyancy effects are important, which combined with a high enough Re ( $\gtrsim 200$ in our study), causes the development of the upstream secondary vortex and the stratification of the flow into two main recirculating cells. As previously observed in earlier studies, for high enough Ri the recirculation is no longer encapsulated, the flow becomes unsteady, and an oscillatory instability develops. This is observed in our simulations starting from Re = 500, Ri = 10. The analysis of the unsteady regime reveals a very rich phenomenology where the geometry of the problem couples with the oscillatory thermal instability. This regime is characterized by the periodic emission of pairs of vortices generated from the upper downstream vertex of the square cavity, and pseudoperiodic variations of the Nusselt number which persist at least up to Re = 1500, while the two main vortices remain in the cavity. Our observations extend previous studies and shed a new light on the characteristics of the oscillatory instability and the role of the Reynolds and Richardson numbers.



## 1. Introduction

The Lattice Boltzmann Method (LBM) has become a powerful alternative to finite element and finite volume methods, for solving different problems and applications in various engineering fields such as different fluid flow cases, transport problems for single and multiphase flow, heat and mass transfer, compressible flows, porous media, magnetodynamics and chemical reaction problems [1]. In this work, LBM is applied to study the problem of mixed convection in an open cavity with a heated bottom wall. This particular case can be found in various engineering applications such as solar devices, heat exchangers, nuclear reactors, electronic systems, etc. Experimental studies of buoyant flow in open cavities can be found in the literature [2,3], and several authors have used LBM in different geometries and complex flows [4–7]. The problem of laminar natural convection in shallow inclined open cavities was addressed in Ref. [8], where a constant heat flux is imposed through the wall facing the opening of the cavity. Mixed convection in cavities has received less attention, but several authors have previously analyzed this problem: in the early '90s, Papanicolaou and Jaluria [9–11] studied the mixed convection in a rectangular enclosure, analyzing the flow for different inlet/outlet and heat source locations as it happens in a cooled electronic device. Manca et al. [12] studied the natural and mixed convection in rectangular cavities (2D) with the T type of geometry and the effect of the position of the heated wall. Stiriba and colls. [13,14] carried out a numerical study of mixed convection for incompressible laminar flow past an open cubical cavity, showing that it exhibits a three-dimensional structure. Abdelmassih et al. also studied numerically [15] and experimentally [16] the same problem. These authors presented results for steady and unsteady laminar regimes in three dimensions, where the effect of the buoyancy force was analyzed for a range of Reynolds and Richardson numbers, observing the effect of natural and forced convection using, in a series of studies, a three-dimensional incompressible finite volume flow solver, ANSYS-FLUENT software and DPIV experimental technique. The bottom of the cavity was heated at constant temperature, while the other walls were adiabatic, at a Prandtl number for the fluid equal to 0.7. Observations indicated that for Reynolds numbers between 100 and 500, the flow was steady with a recirculating cell inside the cavity for all the Richardson numbers studied, but the

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**Nomenclature**

| | |
|---|---|
| $\alpha$ | thermal diffusivity, lattice units |
| $\Delta T$ | temperature differential $=(T_{hot} - T)$ |
| $\mathbf{\Omega}$ | collision matrix operator, MRT model |
| $\nu$ | kinematic viscosity, lattice units |
| $\omega_i$ | weights of the discrete velocities, D2Q9 model |
| $\tau_c$ | dimensionless relaxation time for temperature |
| $\tau_\nu$ | dimensionless relaxation time for momentum |
| $\mathbf{c}_i$ | discrete velocity directions |
| $\mathbf{u}$ | macroscopic velocity |
| $\theta$ | dimensionless temperature in the vertical mid plane of the cavity |
| $c$ | speed on the lattice $=\Delta x/\Delta t$; lattice space and time step size |
| $c_s$ | lattice speed of sound $=c/\sqrt{3}$ |
| $F$ | dimensionless frequency |
| $F_i$ | external force $=3\rho\omega_i g\beta(T - T_{ref})$ |
| $f_i$ | density distribution function |
| $f_i^{eq}$ | equilibrium density distribution function |
| $g_i$ | internal energy distribution function |
| $g_i^{eq}$ | equilibrium internal energy distribution function |
| $G_r$ | Grashof number $=g\beta(T_{hot} - T_{ref})L^3/\nu^2$ |
| $L = H$ | length and height of the cavity |
| $Nu$ | local Nusselt number $=-\frac{L}{\Delta T}\frac{\partial T}{\partial y}\|_{wall}$ |
| $Pr$ | Prandtl number $=\nu/\alpha$ |
| $Re$ | Reynolds number $=u_0 L/\nu$ |
| $Ri$ | Richardson number $=G_r/Re^2;\ g\beta(T_{hot} - T_{ref})L/u_0^2$ |
| $T$ | local temperature |
| $T_c$ | cold temperature, lattice units |
| $T_h$ | hot temperature, lattice units |
| $T_{hot}$ | hot temperature |
| $T_{ref}$ | reference temperature |
| $u_0$ | characteristic velocity of the flow |
| $U_x$ | dimensionless $x$-component of the velocity in the vertical mid plane |
| $U_y$ | dimensionless $y$-component of the velocity in the horizontal mid plane |
| $U_{lbm}$ | lattice velocity |
| $\overline{Nu}$ | average Nusselt number $=\frac{1}{L}\int_0^L Nu\,dx$ |

flow turned unsteady between Re = 500 and 1000 for Ri $\geqslant$0.01. For Richardson numbers exceeding ~0.01, heat transfer was enhanced for all Reynolds numbers: the higher the Ri, the higher the Nusselt number, as the buoyancy force became stronger and natural convection was dominant. This had the effect to push the center of the recirculating cell towards the upper right part of the cavity.

In the present study, we consider a similar arrangement with an open square cavity embedded in the bottom wall of a channel and present a detailed analysis of the two dimensional problem, which differs from the three dimensional case in several aspects. It also differs from the results obtained by previous authors [9–12] due to the fact that the T geometry has a great influence in the flow and the instabilities observed. We use the thermal LBM approach, combined with ANSYS-FLUENT simulations to validate our findings. Using the same T configuration as in the 3D case, we will show how in 2D and above a certain value of the Richardson number, the recirculating cell is split into two cells and that an unsteady flow regime is found exhibiting an intermittent pattern characterized by the periodic emission of hot plumes towards the channel outlet, originating from the upstream vertex of the cavity. The formation of a double recirculation and the development of the intermittent regime has not been previously reported in the square or cubic cavity under similar conditions.

Likewise the Nusselt number shows a complex and pseudo-periodic behavior where the values oscillate in a wide range. This oscillating thermal instability was also found by Papanicolaou and Jaluria [9], who showed that, for a given Reynolds number (Re = 100) there exists a critical Richardson number (Ri = 32), above which an oscillating regime sets up for the outflow heat flux. The T shape of the geometry studied in our case, renders a more complex and different problem, and a richer phenomenology. Remarkably, Zamzari et al. [17] studied this very problem with the same two-dimensional geometry, but such unsteady regimes could not be identified in their study, due to the fact that the range of Reynolds numbers (200–500) and Richardson numbers (0.25–1) considered was too restrictive.

In the following sections we will describe the physical system, the numerical setup and the details of the method used for the analysis, to end up with the results and discussion of the work.

## 2. Physical problem

As shown in Fig. 1, the model consists of an open square cavity of side $L = 0.1$ m with a bottom wall heated at a constant temperature $T_{hot}$. Past the cavity, the channel is of length $3L$, in order to minimize the effect of recirculation in some cases. For the inlet flow and the top wall, we fixed their respective temperatures $T_{ref}$ at the value 298.1450 K (room temperature), and an uniform inlet velocity $u_0$ was set. The choice of velocity profile at the inlet mimicks the flow from a convergent nozzle, which is relatively easy to reproduce experimentally. Moreover, an inlet boundary condition where a fully developed velocity profile is set does not alter the features of the flow and the results obtained are similar. The values of $T_{hot}$ and $u_0$ were varied to give the different cases for the Reynolds $(50, 100, 200, 400, 600, 1000)$ and Richardson numbers $(0.01, 0.1, 1, 10)$ considered. Non-slip and adiabatic boundary conditions were applied at all the other boundaries; an open boundary condition was used for the flow at the outlet to avoid reverse flow.

## 3. Numerical method

### 3.1. Lattice Boltzmann equation

The Lattice Boltzmann Method (LBM) is an alternative and computationally convenient method for the simulation of fluid flow, having as predecessor the lattice gas automata (LGA) [18], which

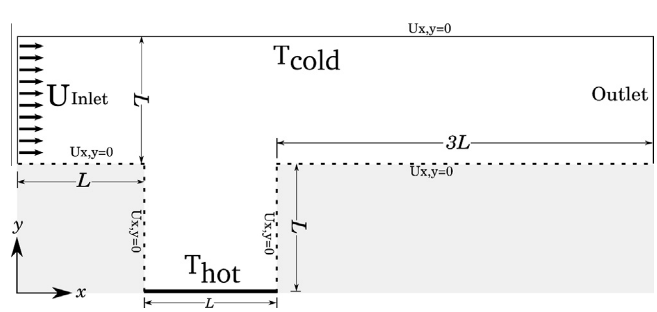


**Fig. 1.** Illustration of open cavity with boundary conditions.