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Compressibility effect on natural convection flow along a vertical plate with isotherm and streamwise sinusoidal surface temperature

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ABSTRACT

In the present investigation compressibility effect on two-dimensional steady natural convection boundary layer flow of a viscous fluid past a vertical plate has been studied. The governing equations are transformed into a non-dimensional form and the solutions are obtained by four different methods, namely, a perturbation method for the small values of local compressibility variable, ξ, asymptotic solutions for large ξ, the direct finite difference method and the implicit finite difference method with Keller-box scheme for all ξ. In order to examine the results obtained from the four methods, calculations have been carried out for a wide range of parameters. The results obtained by the above mentioned four methods are compared and found in excellent agreement. Moreover, the effects of various practical values of the relative temperature difference, α, and the surface temperature wave amplitude a on the local skin friction, the rate of heat transfer and the isotherms are shown graphically.

1. Introduction

The theory of compressible boundary layer flow is of great interest in many mechanical and aerospace engineering practice and the effect of compressibility on a flow is also important for many industrial and engineering applications, such as the design of high speed aircraft, gas turbines, stream turbines, reciprocating engines, natural gas transmission lines and combustion chambers. Because of safety and noise considerations, in high speed ground transportation system the attenuation of a traveling shock has become important. From the quasi-steady state compressible boundary layer flow, Moore [1] developed the first-order deviations of the velocity and temperature profiles, and he also studied the physical parameters which govern the behavior of an unsteady boundary layer with fluctuating pressure gradient. Since then, the theory of compressible boundary layer flow has been much refined and generalized. There are many researchers who have studied the compressible boundary layer flow. For example, Stewartson [2] studied the theory of laminar boundary layers in compressible fluids. Asymptotic approach has been applied to study the laminar flow of compressible fluid having variable fluid properties by Gross and Dewey [3] and Herwig [4] in the boundary layer region. The effect of a sharp pressure rise on a compressible laminar boundary layer flow was studied by Curle [5]. Van Oudheusden [6] also presented a complete Crocco integral method for two dimensional laminar boundary layer flow over an adiabatic wall. Gersten and Herwig [7] and Schlichting and Gersten [8] presented a good description about the compressible boundary layer flow in their books. Recently, Hossain and Pop [9,10] have investigated the effect of heat transfer on compressible boundary layer flow past a sphere and over a circular cylinder. Davies and Walker [11] have studied the solutions of the compressible laminar boundary-layer equations and their behavior near separation. A class of compressible laminar boundary-layer flows and the solution behavior near separation was investigated by Antonios Liakopoulos and Chen-Chi Hsu [12]. Semi-similar solution of an unsteady compressible three-dimensional stagnation point boundary layer flow with massive blowing was obtained by Vasantha and Nath [13]. Krishnaswamy and Nath [14] also studied the compressible second-order boundary layers for three-dimensional stagnation point flows with mass transfer.

Thermally driven convection of compressible fluids occurs in many branches of engineering applications. The fluid may be heated as a result of chemical reaction within the flow. And as in transportation of a cryogenic fluid, it may be heated directly when comes into sudden contact with a surface at room temperature. In all these applications, when the rate of heat transfer is large, the thermally induced motion has significant effects and interacts in

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an important way with other phenomena. There are many technological processes in which this thermally induced motion plays an important role. For example, cryogenic fluid storage and transfer systems experience large and sudden temperature changes in transfer lines and in storage containers during filling. The strong acoustic oscillations of combustion systems can effect the fluid and possibly degrade performance of the system.

Various papers have been published which deal with the natural convection flow. Schmidt and Beckmann [15] presented the formulation of the free convection problem for flow along a vertical plate with uniform temperature. The problem of the free convection flow about a flat plate oriented in direction parallel to that of the generating body force under the prime assumption that the relative temperature difference is small was provided by Ostranch [16]. Hara [17] was the pioneer of the investigation of natural convection flow with the effects of variable property by the perturbation analysis for air. But the solution is applicable for small values of perturbation parameter α, the relative temperature difference. The effect of variable properties and variable wall temperature on the nature convection was also considered by Sparrow [18]. At the same time, natural convection in a gas with variable viscosity was studied by Tanaev [19]. Sparrow and Gregg [20] have also conducted a study of natural convection from an isothermal vertical surface for gases and liquid mercury with variable properties. But they assumed in their formulation that the flow is essentially and that the compressibility terms in the energy equation are negligible. Hara [21] extended the range of application of his previous perturbation solution to $\alpha = 2$ and 4 by a successive approximation method. Later, Eshghy and Morrison [22] studied the compressibility effect on the free convection flow. They provided a more complete treatment of this problem by introducing two parameters, namely, the relative temperature difference and the ratio of the specific heats at constant pressure and volume. The mathematical forms of viscosity variation with temperature which result in similarity solutions for laminar natural convection from a vertical isothermal surface in liquids with temperature dependent viscosity were shown by Carey and Mollendorf [23]. Moulic and Yao [24] have studied the free convection from vertical surface held at uniform temperature with the effect of streamwise surface undulations. Takhar et al. [25] provided the series solutions of free convection heat transfer due to the simultaneous action of buoyancy, radiation and transverse magnetic filed past a semi-infinite vertical plate. Hossain [26] investigated the effect of a fluctuating surface temperature and surface concentration on surface heat flux and surface mass flux from a vertical flat plate. Hossain and Roy [27] also studied the effects of streamwise temperature and species concentration variations on a steady natural convection flow from a vertical plate.

Laminar two unsteady mixed-convection boundary layer flow of a viscous incompressible fluid past a sharp wedge has been studied by Hossian et al. [28]. They solved the governing equations by perturbation method for small time, asymptotic solutions for large time, and the implicit finite difference method (FDM) with Keller-box scheme for all time. Dinarvand et al. [29] obtained the similarity solution for the unsteady laminar incompressible boundary layer flow of viscous electrically conducting fluid in stagnation point region of an impulsively rotating and translating sphere with a magnetic field and a buoyancy force by the homotopy analysis method (HAM). Rashidiet al. [30] got the numerical solution of steady viscous supersonic axisymmetric flow with a diagonal fourth-order finite difference method. Chamkha et al. [31] investigated the effect of uniform lateral mass flux on non-Darcy natural convection of non-Newtonian fluid along a vertical cone embedded in a porous medium filled with a nanofluid. They got the solutions of the governing equations by implicit finite difference method together with Keller box method.

In the present paper we propose to investigate the natural convection flow of viscous compressible fluid along a vertical plate, that posed by Eshghy and Morrison [22]. Since these authors investigated this model for smaller value of local compressibility variable ξ (=$2\gamma g x/c_1^2$, where γ is the ratio of the specific heats, c_1 is the sound speed, g is the gravity and x is axial distance from the leading edge). In the present investigation asymptotic solutions of the governing reduced equations for small and large values of ξ. Solutions of the system of equations are also obtained for all ξ region employing two methods; namely, the direct finite difference method as well as the implicit finite difference method together with Keller-box elimination technique. Further we also investigate the same problem taking into account the effect of streamwise sinusoidal surface temperature variation. The results are obtained in terms of the local skin friction coefficient, the local heat transfer coefficient and the isotherms with effect of the temperature difference parameter, α, and amplitude oscillation, a, of the surface temperature, for fixed Prandtl number, $\Pr = 0.7$, and $\gamma = 1.4$ which appropriate for all diatomic gases like air, helium, hydrogen, etc.

2. Formulation of the problem

We consider two-dimensional steady free convection flow of a viscous compressible fluid along a vertical plate with isothermal and fluctuating surface temperature that varies in the axial direction. The flow configuration and the coordinate systems are shown in Fig. 1. Under the usual boundary layer assumptions the full set of governing equations, namely, the continuity equation, momentum equation and energy equation are written as follows:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\left(\frac{\rho_1 - \rho}{\rho}\right) + \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \tag{1}$$

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = -\frac{T}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p u\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

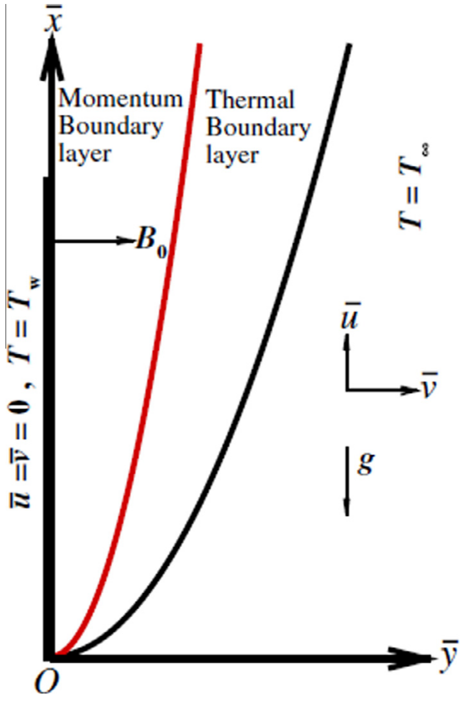

Fig. 1. Schematic diagram of the physical model and the coordinate system.