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Free convection in a trapezoidal cavity filled with a micropolar fluid

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ABSTRACT

The present investigation deals with the study of steady laminar natural convective flow and heat transfer of micropolar fluids in a trapezoidal cavity. The bottom wall of the cavity is kept at high constant temperature, the inclined walls is kept at low constant temperatures while the top horizontal wall is adiabatic. Governing equations formulated in dimensionless stream function and vorticity variables has been solved by finite difference method of the second order accuracy. Comprehensive verification of the utilized numerical method and mathematical model has shown a good agreement with numerical data of other authors. Computations have been carried out to analyze the effects of Rayleigh number, Prandtl number and vortex viscosity parameter both for weak and strong concentration cases. Obtained results have been presented in the form of streamlines, isotherms and vorticity profiles as well as the variation of the average Nusselt number and fluid flow rate. It has been shown that an increase in the vortex viscosity parameter leads to attenuation of the convective flow and heat transfer inside the cavity.

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1. Introduction

Micropolar fluid is a subject of microphoric fluid theory. The detailed description and the modeling of micropolar fluids were initially introduced by Eringen [1–3]. This theory has become very important to engineers and scientists working with hydrodynamic-fluid problems and phenomena for the last few decades. The potential importance of micropolar fluids in industrial applications has motivated many researchers to extent the study in numerous ways to include various physical effects. The essence of the theory of micropolar fluid flow lies in the extension of the constitutive equations for Newtonian fluids so that more complex fluids such as particle suspensions, liquid crystals, animal blood, lubrication, colloidal suspensions, turbulent shear flows, etc. can be described by this theory. In practice, the theory of micropolar fluids requires that one must add a transport equation representing the principle of conservation of local angular momentum to the usual transport equations for the conservation of mass and momentum, and additional local constitutive parameters are also introduced. The special features of micropolar fluids were discussed in two comprehensive review papers of the subject and application of this theory by Ariman et al. [4,5] and in the books by Eringen [6] and Łukasewicz [7].

The convective motion driven by the buoyancy forces is a well-known natural phenomenon and has attracted interest of many researchers. In particular, the topic of natural convection in cavities has received much attention in the past since many practical heat transfer situations can be modeled as flows in cavities. There have been numerous investigations of natural convective heat transfer that occurs in an enclosure (Sathiyamoorthy et al. [8], Kandaswamy and Nithyadevi [9], etc.). Sathiyamoorthy et al. [8] presented the numerical study of steady natural convection in a closed square cavity under different boundary conditions. They showed that for small Rayleigh numbers the average Nusselt number was almost constant due to heat conduction and increased steadily as *Ra* increased.

Natural convection heat transfer and fluid flow were studied for trapezoidal enclosures filled with a viscous (Newtonian fluid) mostly at differentially heated temperature boundary conditions, see Moukalled and Darwish [10,11], Boussaid et al. [12], Moukalled and Darwish [13], Kuyper and Hoogendoorn [14], Sadat and Salagnac [15]. Analysis of convective heat transfer and fluid flow of micropolar fluid in a vertical channel, lid-driven cavity and square cavity has been conducted in [16–19]. We mention also to this end, the paper by Hsu and Hong [20] on natural convection of micropolar fluids in an open cavity, which consists by two adiabatic horizontal walls and one heated vertical wall, while the open end has several different geometric features. However, to our best knowledge, trapezoidal enclosures filled with a micropolar fluid

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Nomenclature

g	gravitational acceleration
H	height of the cavity
j	micro-inertia density, $j = L^2$
K	vortex viscosity parameter, $K = \kappa/\mu$
L	length of the bottom hot wall
N	dimensionless microrotation
$\bar{N}$	dimensional microrotation
Nu	local Nusselt number
$\overline{Nu}$	average Nusselt number
p	pressure
Pr	Prandtl number, $Pr = \nu/\alpha$
Ra	Rayleigh number, $Ra = g\beta(T_h - T_c)L^3/(\alpha\nu)$
T	temperature of the fluid
T_c	temperature of the cooled walls
T_h	temperature of the hot wall
$\bar{u},\ \bar{v}$	dimensional velocity components along $\bar{x}$ and $\bar{y}$ coordinates
$u,\ v$	dimensionless velocity components along x and y coordinates
$\bar{x}$	dimensional coordinate measured along the bottom wall of the cavity
$\bar{y}$	dimensional coordinate measured along the vertical direction of the cavity
x, y	dimensionless Cartesian coordinates

Greek symbols

α	thermal diffusivity
β	volumetric expansion coefficient of the fluid
γ	spin-gradient viscosity, $\gamma = (\mu + \frac{\kappa}{2})j$
θ	dimensionless temperature
κ	vortex viscosity
μ	dynamic viscosity
ρ	fluid density
φ	inclination angle of the inclined cooled walls
ψ	dimensionless stream function
ω	dimensionless vorticity

have not been considered yet. Therefore, the main objective of this paper is to examine the natural convection in a trapezoidal cavity filled with a micropolar fluid. Streamlines, isotherms, average Nusselt number and fluid flow rate are presented and discussed in details.

2. Basic equations

The domain of interest filled with a micropolar fluid is presented in Fig. 1 with dimensional Cartesian coordinates $\bar{x}$ and $\bar{y}$. The trapezoidal enclosure is bounded by isothermal inclined cooled walls of temperature T_c, isothermal bottom hot wall of temperature T_h ($T_h > T_c$) and adiabatic top wall. All four walls of the cavity are assumed to be rigid and impermeable.

The micropolar fluid is considered to satisfy the Boussinesq approximation and the flow regime is laminar. Taking into account the theory of Eringen [1–3] for the micropolar fluid flow the governing equations can be written in dimensional Cartesian coordinates as follows

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{1}$$

$$\rho\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) = -\frac{\partial \bar{p}}{\partial \bar{x}} + (\mu+\kappa)\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) + \kappa\frac{\partial \bar{N}}{\partial \bar{y}} \tag{2}$$

$$\rho\left(\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}}\right) = -\frac{\partial \bar{p}}{\partial \bar{y}} + (\mu+\kappa)\left(\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right) - \kappa\frac{\partial \bar{N}}{\partial \bar{x}} + \rho\beta(T - T_c)g \tag{3}$$

$$\rho j\left(\bar{u}\frac{\partial \bar{N}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{N}}{\partial \bar{y}}\right) = \gamma\left(\frac{\partial^2 \bar{N}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{N}}{\partial \bar{y}^2}\right) + \kappa\left(\frac{\partial \bar{v}}{\partial \bar{x}} - \frac{\partial \bar{u}}{\partial \bar{y}}\right) - 2\kappa\bar{N} \tag{4}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha\left(\frac{\partial^2 T}{\partial \bar{x}^2} + \frac{\partial^2 T}{\partial \bar{y}^2}\right) \tag{5}$$

In order to analyze the fluid flow and heat transfer in general scale we introduce the following dimensionless variables

$$x = \bar{x}/L, \quad y = \bar{y}/L, \quad u = \bar{u}/\sqrt{g\beta(T - T_c)L}, \quad v = \bar{v}/\sqrt{g\beta(T - T_c)L},$$
$$\theta = (T - T_c)/(T_h - T_c), \quad N = \bar{N}/\sqrt{g\beta(T - T_c)/L} \tag{6}$$

and also stream function ψ $\left(u = \frac{\partial\psi}{\partial y}, v = -\frac{\partial\psi}{\partial x}\right)$ and vorticity $\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$. Therefore the governing Eqs. (1)–(5) using the dimensionless variables (6) can be written as follows

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega \tag{7}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \omega}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \omega}{\partial y} = (1+K)\sqrt{\frac{\Pr}{Ra}}\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right) - K\sqrt{\frac{\Pr}{Ra}}\left(\frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2}\right) + \frac{\partial \theta}{\partial x} \tag{8}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial N}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial N}{\partial y} = \left(1 + \frac{K}{2}\right)\sqrt{\frac{\Pr}{Ra}}\left(\frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2}\right) + K\sqrt{\frac{\Pr}{Ra}}(\omega - 2N) \tag{9}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y} = \frac{1}{\sqrt{Ra \cdot \Pr}}\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) \tag{10}$$

with the following boundary conditions

$$\begin{aligned}
&\psi = 0, \quad \omega = -\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial y^2},\\
&N = n\cdot\omega, \quad \theta = 0 \quad \text{on left and right inclined walls}\\
&\psi = 0, \quad \omega = -\frac{\partial^2 \psi}{\partial y^2}, \quad N = n\cdot\omega, \quad \theta = 1 \quad \text{on bottom wall}\\
&\psi = 0, \quad \omega = -\frac{\partial^2 \psi}{\partial y^2}, \quad N = n\cdot\omega, \quad \frac{\partial \theta}{\partial y} = 0 \quad \text{on top wall}
\end{aligned} \tag{11}$$

Here $n (0 \leqslant n \leqslant 1)$ is a micro-gyration parameter with $n = 0$ corresponding to the case where the particle density is sufficiently great that microelements close to the wall are unable to rotate

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