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Porosity distribution optimization of insulation materials by the variational method

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ABSTRACT

Heat transfer performance of thermal insulation materials is determined by many independent parameters, it is hard to optimize the internal structures of such materials through constructing a mathematical relation between the independent parameters and the optimization objective. In this paper, we theoretically analyze such factors as temperature, pressure and porosity influencing the heat transfer performance of porous materials, establish a mathematical model by using the entransy theory with such constraints as fixed mass and thickness of the porous material, and finally apply the variational principle to derive the governing equations for optimizing the porosity/solid fraction distribution in porous materials. Meanwhile, a 1-D and a 2-D physical model are taken as examples to show the applications. When the surface temperatures as well as the total mass and thickness of the high-porous structure are given, we get the optimal porosity distribution through solving the newly derived governing equations. The results show that both heat flux and the effective thermal conductivity of the optimized structure is the minimal. That is, the thermal insulation performance is optimal.

1. Introduction

High-porosity thermal insulation materials are widely employed in various industrial applications, such as building insulations, heat exchangers and thermal protection systems [1,2], where the thermal insulation performance of the material is the most important. Heat transfer processes in a thermal insulation material include solid conduction, gas conduction and thermal radiation, which are all influenced by the complex internal structure and the properties of the material. Especially, when the insulation materials are used in a high temperature environment, up to 1000 °C, thermal radiation becomes more significant [3]. In these cases, the complex internal structure, the properties of the materials and the high temperature environment make the analysis and the optimal design of thermal insulation performance more complex and difficult [4].

Many researchers studied the effective thermal conductivities of thermal insulation materials by experimental and theoretical methods. For instance, Liang and Qu [5] analyzed the local equivalent and effective thermal conductivity of porous materials with different air cavities. Arulanantham et al. [6] investigated the combined radiative and conductive heat transfer processes across a honeycomb insulation material by using an exponential kernel approximation. Spinnler et al. [7] and Daryabeigi [8,9] established several experimental facilities to test the effective thermal conductivities at different environmental pressures and temperatures, and evaluated the analytical models through the experimental data. Besides, some other authors focused on the optimal design of porous materials to improve the thermal insulation performance. For instance, Venkataraman et al. [10] optimized the density profiles under one-dimensional steady-state condition and showed that the functional grading of foam density can effectively reduce the heat transfer rate at given foam thickness. Zhu et al. [11] reduced the maximum temperature of an insulation structure under transient conditions by using the method proposed by Venkataraman et al. [10]. Zhao and Lu [12] optimized the volume fraction to minimize the weight of thermal protection system under some constraints by applying the single-parameter analysis method.

The above mentioned studies mainly focused on analysis of the heat transfer mechanisms by using different models, the test of the effective thermal conductivities of different materials, and the optimization of the material internal structures to improve the thermal insulation performance. However, because the thermal insulation performance is determined by many independent parameters, it is difficult to obtain the optimal solution by the single parameter analysis method. There is still lacking a general mathematical method to optimize the thermal insulation perfor-

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Nomenclature

A	the Lagrange multiplier
B	the Lagrange multiplier
c	heat capacity, J kg^{-1} K^{-1}
D_f	effective fiber diameter, m
d_g	gas collision diameter, m
e	specific extinction coefficient, m^2 kg^{-1}
f	solid fraction
J	the functional
k	thermal conductivity, W K^{-1} m^{-1}
K_B	Boltzmann constant, J K^{-1}
L	insulation structure thickness, mm
L_c	gas conduction characteristic length, m
P	pressure, Pa
Pr	gas Prandtl number
q	heat flux W m^{-2}
T	temperature, K
x	the coordinate, mm
α	thermal accommodation coefficient
β	extinction coefficient, m^{-1}
γ	specific heat ratio
λ	gas mean free length, m
ρ	material density, kg m^{-3}

Subscripts

f	solid fraction
g	gas
H	high temperature
L	low temperature
m	matrix material
r	radiation
s	solid
c	conductivity
opt	optimal

mance with more complex constraints. In recent years, for the optimization of heat conduction, heat convection and thermal radiation, Chen et al. [13–15] and Meng et al. [16] introduced the variational method into heat transfer optimization by constructing a Lagrange function, and optimized the volume-point heat conduction process [14], the laminar heat transfer process in round tube [15], and the turbulent heat transfer process between two parallel plates [16].

Inspired by the application of the variational method in heat transfer optimization, the influence of several independent parameters, e.g. temperature, pressure and solid fraction, on the heat transfer performance of porous materials are analyzed, a Lagrange function is constructed by using the entransy concept together with such constraints as fixed mass and thickness of the porous material, and then the variational method is applied to derive the governing equations for the porosity distribution optimization in the porous material. Finally, the porosity distribution of two physical models are optimized to show the potential applications of the newly proposed method.

2. Heat transfer analysis in insulation materials

In fibrous insulation materials, heat is transferred by four different kinds of mechanisms: gas conduction and natural convection in the void pores between fibers, thermal radiation between fibers, and solid conduction through fibers, where natural convection is neglected in this study [4]. Fig. 1 gives the physical model of 1-D fibrous insulation sample, which is used to analyze the heat transfer mechanisms for simplicity. The surface temperatures of the bottom and the top plates are T_L and T_H, respectively, and L is the thickness of the insulation material.

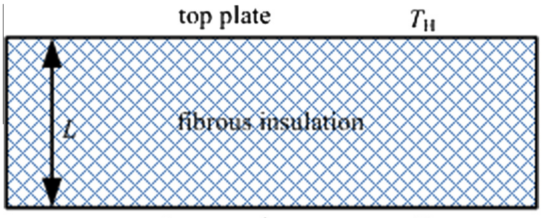

Fig. 1. The schematic diagram of a fibrous insulation sample.

The energy conservation equation for the fibrous insulation sample is [17]

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k_c \frac{\partial T}{\partial x}\right) - \frac{\partial q_r}{\partial x}, \tag{1}$$

where, T is temperature, t is the time, ρ is the density of insulation material, c is the heat capacity, and k_c is the equivalent thermal conductivity, which includes the gas conduction and the solid conduction. q_r is the radiation heat flux. In order to analyze the heat transfer performance of a porous insulation material, it is required to identify such two terms as the equivalent thermal conductivity and the radiation heat flux in Eq. (1).

2.1. Gas conduction and solid conduction

The gas conductivity in porous media always varies with temperature and pressure, which is calculated based on the temperature jump theory by using the following equation [4]:

$$k_g = \frac{k_g^*}{1 + 2\frac{2-\alpha}{\alpha}\left(\frac{2\gamma}{\gamma+1}\right)\frac{1}{Pr}\frac{\lambda}{L_c}}, \tag{2}$$

where α is the thermal accommodation coefficient, γ is the specific heat ratio, k_g^* is the thermal conductivity of gas at atmospheric pressure, and Pr is the gas Prandtl number. k_g^* and Pr are expressed as [18]

$$k_g^* = 3.954 \times 10^{-3} + 7.7207 \times 10^{-5} T - 1.6082 \times 10^{-8} T^2, \tag{3}$$

$$Pr = 0.7086 - 3.7245 \times 10^{-6} T + 2.2556 \times 10^{-10} T^2. \tag{4}$$

In Eq. (2), λ is the gas mean free length, which is a function of temperature and pressure,

$$\lambda = \frac{K_B T}{\sqrt{2}\pi d_g^2 P}, \tag{5}$$

where K_B is the Boltzmann constant, d_g is the gas collision diameter, L_c is the gas conduction characteristic length, which is calculated by

$$L_c = \frac{\pi}{4}\frac{D_f}{f}, \tag{6}$$