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Velocity reversals via bifurcation in thermal convection

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ABSTRACT

This numerical study reveals and explains a mechanism of eddy emergence in the steady two-dimensional thermal convection. A thin horizontal container is filled with water. The bottom and top walls are adiabatic while the sidewalls have prescribed temperatures. Gravity and a horizontal gradient of temperature drive the water circulation from the cold end to the hot end near the bottom and back near the top. As the flow strength, characterized by the Grashof number *Gr*, increases, the horizontal velocity reverses and local circulation cells emerge via bifurcation near the central stagnation point. We argue that the reversals are likely caused by the entrainment effect of jets, which form near the horizontal walls. This explains the experimental observations of Kirdyashkin (1984). No instability develops for $Gr \leqslant 10^7$ due to the stable vertical stratification of density. The obtained results are of fundamental interest and can be relevant for the development of efficient heat exchangers.

1. Introduction

1.1. Review of prior studies

The thermo-gravitational convection in a horizontal layer of a fluid induced by the horizontal gradient of temperature is one of basic problems of heat transfer. The global circulation between the equatorial and polar oceanic regions occurs mostly due to the horizontal gradient of temperature [1]. Similar flows are observed in shallow water pools used for the removal of waste heat and in the technology of crystal growth [2]. More applications are related to cooling systems for nuclear reactors and solar energy collectors [3].

This problem also is of fundamental interest being a unique case where experimental, analytical and numerical results allow meaningful comparison in a wide range of the flow strength, characterized by the Grashof number, *Gr*.

Accordingly, the problem has attracted the attention of many researchers. The detailed reviews [3–6] cover earlier studies. Below, we discuss works which are relevant for this problem, but not mentioned in these reviews.

For small and moderate *Gr*, the central part of elongated flow is described by the elegant polynomial solution to the Boussinesq equations found by Ostroumov [7]: $u/u_{\max} = (y^3 - y)(27)^{1/2}/2$, where u is the horizontal velocity and $u_{\max}$ is its maximal magnitude; y is the vertical coordinate, divided by the layer half-width. The horizontal walls are located at $y = \pm 1$. Birikh [8] generalized the solution [7] for the case where the upper surface is free and subject to the thermal surface-tension (Marangoni) effect.

For small and moderate *Gr*, the solutions [7,8] excellently agree with the experimental measurements performed by Kirdyashkin [2]. Kirdyashkin observed the steady flow up to $Gr \approx 10^7$. As *Gr* increases, the flow in the container transforms from that described by solution [7] to the boundary-layer pattern with jets developed near the horizontal walls and a slow double-reversed flow in between the jets. It is striking that no instability occurs despite the $u(y)$ profile becomes wavy with a few inflection points. This experimental fact seems contradicting to the stability studies [9–11] of solution [7].

Birikh [9] considered the stability of flow [7] at the Prandtl number $Pr = 0$ and found that the critical value of the Grashof number is $Gr^* = 495$. Gershuni, Zhukhovitsky and Myznikov explored the stability of flow [7] at $Pr > 0$ with respect to two-dimensional [10] and three-dimensional [11] disturbances. They predicted two kinds of instability: (a) shear-layer K-instability related to the existence of inflection point in the $u(y)$ profile and (b) thermal R-instability caused by the unstable density stratification near the horizontal walls; here K is for Kelvin and R is for Rayleigh. As *Pr* increases, the K-instability disappears for $Pr > 0.5$, but the R-instability occurs for any large *Pr*.

Our recent study [12] explains this seeming controversy. The horizontal walls have prescribed temperatures in the stability studies [10,11] while the walls are adiabatic (no heat flux) in the experiment [2]. It was found that the R-instability disappears if the boundary conditions change from the fixed-temperature to

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the adiabatic ones [12]. The physical reason is that the density stratification becomes stable in the entire flow domain for the adiabatic conditions. This explains why Kirdyashkin observed no instability [2].

The analytical solution [7] was generalized for the cylindrical geometry and the centrifugal force replacing the gravity. For a small axial gradient of temperature, the centrifugal convection in a rotating pipe is also described by the polynomial solution [13]: $w/w_0 = 1 - 4r^2 + 3r^4$ and $(T - T_1)/(T_0 - T_1) = (1 - r^2)^3$; where r is the distance from the axis divided by the pipe radius. Subscripts 0 and 1 denote values of axial velocity w and temperature T at the axis and sidewall respectively. There is also analytical (though not polynomial) solution for a gap between two co-rotating pipes [13]. For a narrow gap, the cylindrical problem becomes close to that for a horizontal layer [7]. Birikh and Pukhnachev [14] generalized the solution [13] to describe a two-fluid thermal convection with the Marangoni effect taken into account. The numerical simulations of the air–water centrifugal convection in a cylindrical container [15] agree with the analytical solutions and revealed the emergence of new flow cells as Gr increases.

Surprisingly, no numerical study has been performed for the double-reversed water flow experimentally revealed by Kirdyashkin [2]. The current paper partially fills this gap. Our numerical results agree with the analytical solutions for small and moderate Gr and also agree with Kirdyashkin's experimental measurements for large Gr where $u(y)$ and $T(y)$ significantly differ from those described by the analytical solutions. For convenient comparison, we focus on Gr values which correspond to the figures in Ref. [2].

Our numerical study reveals the following important features which were not reported in Kirdyashkin's paper [2]: (*i*) the development of the boundary layers near the vertical walls where jets form (which are the most high-speed flow parts), (*ii*) the emergence of local circulation cells via bifurcation and (*iii*) the formation of near-stagnation region in the central part of the container resulting in that the most of heat is transported by a thin ring-like jet adjacent to the container boundary. Features (*i*) and (*iii*) agree with prior analytical and numerical results while feature (*ii*) is absolutely new. We provide physical reasons for features (*i*)–(*iii*).

1.2. Flow reversal due to jet entrainment

Since the density stable stratification opposes the fluid vertical motion, the upflow (downflow) focuses near the hot (cold) sidewall and the vertical jets develop even for moderate Gr. As known, a jet entrains an ambient fluid. Schlichting [16] found for a round jet, that the fluid flow rate (through a normal-to-jet plane) increases proportionally to the distance from the jet source and therefore the far-field flow consists mostly of the entrained fluid. If a jet issues from a wall, the flow reversal occurs as Fig. 1 (experiment by Zauner [17]) and Fig. 2 (theory by Schneider [18]) illustrate: the jet goes away from the wall while the entrainment flow moves toward the wall. The reversal also occurs in a plume [19], a flow driven by the thermal surface tension (Marangoni) effect [15] and in a flow near a hot vertical wall [20]. More jet-induced counterflows are discussed in Ref. [21]. These examples indicate that the velocity reversal can be a result of jet-like flow independent of how it is driven.

1.3. Circulation cell due to velocity reversal

The development of local circulation regions is typical of thermal convection and known starting with the Bénard cells [22]. Recent examples are reported in Ref. [15] and paper by Xu et al. [23]. However, the jet entrainment mechanism of the cell

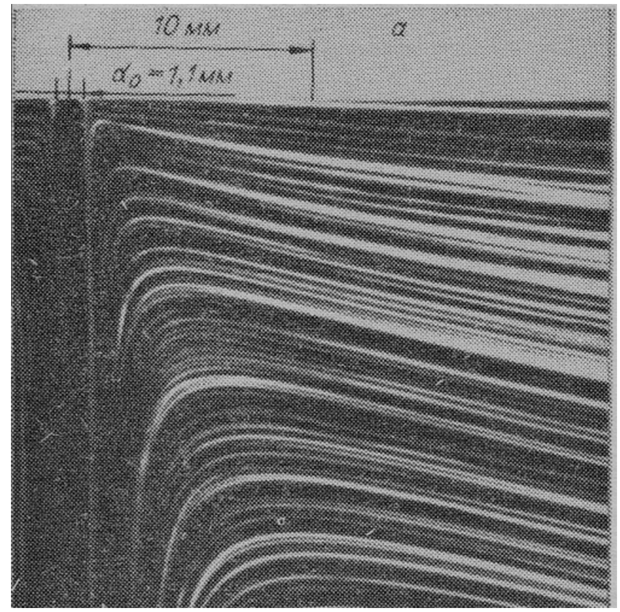

Fig. 1. Visualization shows the velocity reversal in the jet-driven flow [17].

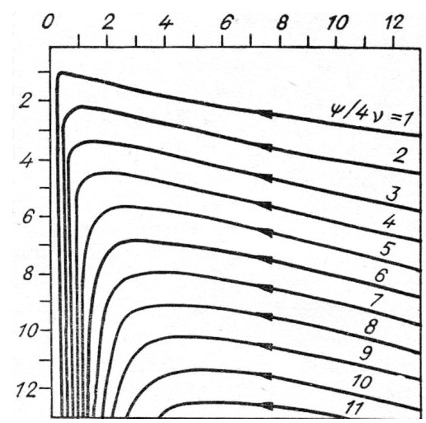

Fig. 2. Explanation of the flow shown in Fig. 1 [18].

formation has not been addressed in the literature. Our paper partially fills this gap by explaining how this mechanism works. To this end, the flow studied by Kirdyashkin's is suitable because it remains stable despite the horizontal velocity becomes double-reversed. This stability seems striking and unique because counterflows typically suffer from the shear-layer K-instability [9–11].

It is shown here that the velocity reversal can results in the emergence of a circulation cell. For this to occur, the reversed velocity magnitude is crucial. No circulation cell is in the jet flow presented in Figs. 1 and 2. In contrast, the Marangoni flow [15] produces circulation cells. Fig. 3 depicts the pattern of air–water centrifugal convection in the sealed cylindrical container rotating around its axis. The bottom (top) disk is cold (hot) and the sidewall is adiabatic.

The air (water) occupies the region $0 < r < 0.5$ ($0.5 < r < 1$). The arrow indicates the flow and Marangoni forcing direction. The light (dark) streamline contours depict to the clockwise (anticlockwise) meridional circulation. With no Marangoni forcing, the water flow is one-cellular. The Marangoni effect causes flow reversals and the development of the three cells of water motion [15].

In Kirdyashkin's case, both the vertical and horizontal near-wall jets likely cause flow reversals, but the horizontal jets only result in