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Unsteady flow of nanofluid with double stratification and magnetohydrodynamics

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ABSTRACT

This article addresses the unsteady flow of viscous nanofluid caused by an inclined stretching sheet. Effects of thermal radiation, viscous dissipation and stratification process due to temperature and concentration are analyzed. Fluid is electrically conducting in the presence of applied magnetic field. The flow consideration is subjected to small magnetic Reynolds number. Induced magnetic field is absent. Appropriate transformations reduce the nonlinear partial differential system to ordinary differential system. Convergent solutions are computed. Interval of convergence is determined. Effects of different parameters on the velocity, temperature and concentration profiles are shown and analyzed. It is concluded that thermal and solutal stratification parameters reduce the velocity distribution. It is also observed that velocity is decreasing function of Hartman number.

1. Introduction

Recently the topic of convective heat transfer through nanoparticles is a popular area of research. Traditional heat transfer fluids such as water, oil and ethylene glycol are poor heat transfer fluids. As the thermal conductivity of these fluids plays an important role in the heat transfer coefficient, so numerous methods have been adopted to enhance the thermal conductivity of these fluids. It is observed that there is enhancement of thermal conductivity by adding nanometer sized particles into the traditional heat transfer fluids. Thus the nanomaterials are recognized more effective in micro/nano electromechanical devices, advanced cooling systems, large scale thermal management systems via evaporators, heat exchangers and industrial cooling applications. Such fluids are very stable with no extra issues of erosion, sedimentation, non-Newtonian properties and additional pressure drop. This is because of tiny size and low volume fraction of nano elements required for thermal conductivity enhancement. Further the canvas of magnetic field has important applications in medicine, physics and engineering. Many equipments such as MHD generators, pumps, bearings and boundary layer control are affected by the interaction between the electrically conducting fluid and a magnetic field. The behavior of the flow strongly depends on the orientation and intensity of the applied magnetic field. The exerted magnetic field manipulates the suspended particles and rearranges their concentration in the fluid which strongly changes heat transfer characteristics of the flow. A magnetic nanofluid has both the liquid and magnetic characteristics. Such materials have fascinating applications like optical modulators, magneto-optical wavelength filters, nonlinear optical materials, optical switches, optical gratings etc. Magnetic particles have key role in the construction of loud speakers as sealing materials and in sink float separation. Magneto nanofluids are useful to guide the particles up the blood stream to a tumor with magnets. This is due to the fact that the magnetic nanoparticles are regarded more adhesive to tumor cells than non-malignant cells. Such particles absorb more power than microparticles in alternating current magnetic fields tolerable in humans i.e. for cancer therapy. Numerous applications involving nanofluids include drug delivery, hyperthermia, contrast enhancement in magnetic resonance imaging and magnetic cell separation. Motivated by all the aforementioned facts, various scientists and engineers are engaged in the discussion of flows of nanofluids [1,2]. Later the boundary layer flows of nanofluids via different aspects have been discussed in the studies [3–21] and many useful attempts therein.

Stratification is an important aspect in heat and mass transfer and it has been studied by several researchers. It arises in the flow fields due to temperature variation, concentration differences or fluids with different densities. Double stratification arises when both heat and mass transfer occur simultaneously. Thermal stratification of reservoirs and oceans, salinity stratification in rivers, estuaries, ground water reservoirs, oceans, heterogeneous mixtures in atmosphere, industrial food and manufacturing processes

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are the few examples of stratification. Density differences in the presence of gravity have a key role on the dynamics and mixing of heterogeneous fluid. For example thermal stratification in reservoirs can reduce the mixing of oxygen to the bottom water to become anoxic through the action of biological processes. Stratification plays important role in lakes and ponds. Because it controls the temperature and concentration differences of hydrogen and oxygen in such environments which may affect the growth rate of various species. Also, the analysis of thermal stratification is important for solar engineering because higher energy efficiency can be achieved with better stratification. Mukhopadhyay and Ishak [22] discussed the mixed convection flow along a stretching cylinder in a thermally stratified medium. Ibrahim and Makinde [23] investigated the effect of double stratification on boundary-layer flow and heat transfer of nanofluid by a vertical plate. Effect of double stratification on MHD free convection in a micropolar fluid has been studied by Srinivasacharya and Upendar [24]. Hayat et al. [25] analyzed the influence of thermal stratification on the radiative flow of Maxwell fluid. Hayat el al. [26] also examined the thermally stratified stagnation point flow of an Oldroyd-B fluid.

Many engineering processes such as fossil fuel combustion energy processes, solar power technology, astrophysical flows and space vehicle re-entry occur at high temperature, so radiative heat transfer plays very important role. Also thermal radiation on flow and heat transfer processes is of major interest in the design of many advanced energy conversion systems operating at high temperature. Thermal radiation effects become important when the difference between the surface and the ambient temperature is large. The Rosseland approximation is used to describe the radiative heat flux in the energy equation. Hamid et al. [27] investigated the effects of radiation, Joule heating and viscous dissipation on MHD Marangoni convection over a flat surface with suction and injection. Ibrahim [28] investigated the effects of mass transfer, radiation, Joule heating, and viscous dissipation on steady MHD Marangoni convection flow over a flat surface with suction and injection. Ali et al. [29] investigated the radiation effects on MHD free convection flow along vertical flat plate in presence of Joule heating and heat generation. Mushtaq et al. [30] studied about thermal radiation effects on the stagnation point flow of upper-convected Maxwell fluid over a stretching sheet. Hayat et al. [31] studied the radiation effects on the flow of Powell–Eyring fluid past an unsteady inclined stretching sheet with non-uniform heat source/sink.

The present work studies the double stratified flow of an electrically conducting nanofluid past an inclined stretching sheet with thermal radiation and viscous dissipation. The problem is first modeled and then solved by homotopy analysis method [32–36]. Convergence region of the derived solutions is determined. The behaviors of Brownian motion and thermophoretic diffusion of nanoparticles have been examined graphically. Discussion relevant to embedded parameters is made using graphical illustration.

2. Model development

Consider the unsteady two-dimensional incompressible flow of nanofluid past a stretching sheet. The sheet makes an angle α with the horizontal direction. The x-axis is taken along the stretching surface in the direction of motion and y-axis is perpendicular to it. Thermal and concentration buoyancy forces are applied to the fluid with double stratified phenomena due to temperature and concentration. The sheet is maintained at temperature $T_w = T_0 + Ax/(1-at)$ and concentration $C_w = C_0 + Dx/(1-at)$. The temperature and mass concentration of the ambient fluid are assumed to be stratified in the form $T_\infty = T_0 + Bx/(1-at)$ and $C_\infty = C_0 + Ex/(1-at)$ respectively (see Fig. 1).

It is assumed that a uniform magnetic field of intensity B_0 acts in the y-direction. The magnetic Reynolds number is assumed to be small so that the induced magnetic field is negligible in comparison with the applied magnetic field. In addition the effects of thermal radiation and viscous dissipation are considered. The continuity, momentum, energy and concentration equations which govern such type of flow are written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\sin\alpha\left[\beta_T(T-T_\infty)(1-C_\infty)+\frac{(\rho^*-\rho)}{\rho}(C-C_\infty)\right]-\frac{\sigma B_0^2 u}{\rho}, \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial T}{\partial y}\frac{\partial C}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]+\frac{\mu}{(\rho C_p)_f}\left(\frac{\partial u}{\partial y}\right)^2+\frac{16\sigma^* T_\infty^3}{3k^*(\rho C_p)_f}\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}. \tag{4}$$

The subjected boundary conditions are

$$u=U=\frac{bx}{1-at},\quad v=0,\quad T=T_w=T_0+\frac{Ax}{1-at},$$
$$C=C_w=C_0+\frac{Dx}{1-at}\ \text{at}\ y=0,$$
$$u\to 0,\quad T\to T_\infty=T_0+\frac{Bx}{1-at},$$
$$C\to C_\infty=C_0+\frac{Ex}{1-at}\ \text{as}\ y\to\infty, \tag{5}$$

where u and v are the velocity components along the x- and y-directions respectively, ν, ρ and σ are the kinematic viscosity, density and electrical conductivity of the fluid, g is the gravitational acceleration, β_T is the coefficient of thermal expansion, T, T_∞, C and C_∞ are the fluid temperature, ambient fluid temperature, fluid concentration and ambient fluid concentration, $\alpha = k/(\rho c)_f$ is the thermal diffusivity, $\tau = (\rho c)_p/(\rho c)_f$ is the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient, k^* is the mean absorption

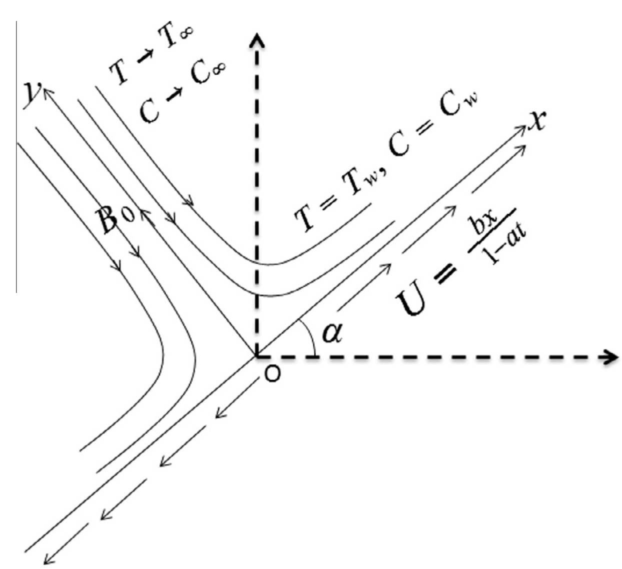

Fig. 1. Geometry of the problem.