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Analytical solutions for thermo-fluidic transport in electroosmotic flow through rough microtubes

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ABSTRACT

The limitations of the microfabrication technology do not allow producing perfectly smooth microchannels. Hence, exploring the influences of roughness on transport phenomena in microtubes is of great importance to the scientific community. In the present work, consideration is given toward the corrugated roughness effects on fully developed electroosmotic flow and heat transfer in circular microtubes. Analytical solutions based on perturbation technique are presented for the problem assuming a low zeta potential under the constant heat flux boundary condition of the first kind. It is revealed that higher values of the corrugation number and relative roughness give rise to smaller Nusselt numbers. Since the same is true for the mean velocity, one may conclude that the roughness effects on the hydrodynamic and thermal features of electroosmotic flow are negative. Further, the Nusselt number is found to be a decreasing function of the Joule heating rate and an increasing function of the dimensionless Debye-Hückel parameter.

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1. Introduction

Electroosmosis is one of the primary electrokinetic phenomena which was discovered more than two centuries ago [1]. It refers to the flow of an ionic solution tangential to an electrified surface under the application of an electric field. Ionic species within a charged interfacial layer (also known as the electrical double layer, EDL) are forced to move by virtue of the external electric field, which in turn actuates the motion of the solvent molecules through viscous interaction, resulting in so-called electroosmotic transport.

The trace of the pertinent literature indicates that the first accurate expression for electroosmotic velocity was derived by Smoluchowski [2] in 1903. Two decades later, Debye and Hückel [3] determined the ionic distribution in solutions of low electrical potential, by means of a linear approximation of the Boltzmann distribution. This paved the way for the development of analytical solutions for electroosmotic flow (EOF) in flat and circular channels by Burgreen and Nakache four decades later [4]. They afterward extended the solutions to account for high surface potentials [5]. Rice and Whitehead [6] investigated fully developed EOF in a narrow cylindrical capillary assuming low zeta potentials. Levine et al. [7] extended Rice and Whitehead's work to high zeta potentials by means of an approximation method.

More recently, Tsao [8] examined the hydrodynamic aspects of EOF in a microannulus, using the Debye-Hückel linearization. His work was extended to high zeta potentials by Kang et al. [9], utilizing an approximate method. Closed form solutions for fully developed EOF in rectangular and semicircular microchannels were presented by Yang [10] and Wang et al. [11], respectively. Xuan and Li [12] developed semi-analytical solutions for electrokinetic flow in microchannels with arbitrary geometry and arbitrary distribution of wall charge.

Despite the hydrodynamic features being well-explored, the study of the thermal aspects of EOF is recent. The pioneering studies in this scope were performed by Maynes and coworkers and include presenting closed form solutions for thermally fully developed EOF in slit and circular channels of small zeta potential with [13] and without [14] considering viscous heating effects. They thereafter extended their works to account for high zeta potentials, albeit numerically [15]. Subsequently, Chakraborty [16] obtained analytical solutions of Nusselt number for thermally fully developed flow in microtubes under a combined action of electroosmotic forces and imposed pressure gradients. Chen [17] performed the same analysis for a slit channel. The viscous heating

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effects on the thermal characteristics of the fully developed mixed flow in slit and circular microducts were analyzed analytically by Sadeghi and Saidi [18] and Yavari et al. [19], respectively. Yavari et al. [20] derived closed form solutions for the hydrodynamic and thermal characteristics of combined electroosmotic and pressure driven flow in a microannulus. Exactly the same, but this time for a rectangular geometry, was done by Sadeghi et al. [21]. The mixed flow characteristics in triangular microchannels were investigated by Liao et al. [22] through a Galerkin-based numerical approach. In a recent study, Vocale et al. [23] paid attention to the problem of EOF heat transfer in elliptical microchannels under axially constant heat flux boundary condition.

The walls of any conduit show some degree of roughness depending on the manufacturing process. Since the surface conditions may affect the flow characteristics to some extent, it is crucial to account for roughness effects in any fluid flow analysis. This is more essential in dealing with microflows, because, as the flow cross section gets smaller the influences of wall phenomena grow significantly. This fact has motivated some researchers to investigate the roughness effects on EOF [24–28]. However, no attention has been given toward the influences of the surface conditions on the associated heat transfer physics, despite recent advancements in using electroosmotic-based micro cooling systems [29–31] that dramatically raise the need for such analyses. The successful utilization of EOF for cooling purposes also draws attention to the thermo-fluidic transport in circular microchannels, as an essential part of microchannel heat sinks [32–35]. In this respect, in spite of considerable attention to the electroosmotic flow and heat transfer in circular microducts, this geometry has been ignored in the studies of the roughness effects. In the present work, the effects of roughness on both the hydrodynamic and thermal features of EOF through circular micropipes are being studied. For a better tractability of the problem, the surface roughness is modeled by considering a corrugated channel surface. The flow is assumed to be both hydrodynamically and thermally fully developed and the thermal boundary condition is assumed to be the constant heat flux of the first kind, H1, which refers to a constant heat flux in the axial direction and a constant temperature in each cross section [36]. Analytical solutions are obtained for the electrical potential, velocity, and temperature fields utilizing the perturbation technique, assuming small amounts of the relative roughness. To the best of our knowledge, this is the first report on analytical solutions for heat transfer in EOFs with surface roughness effects taken into consideration. A complete parametric study is then performed by putting emphasize on the heat transfer aspects and it is shown that roughness has unfavorable effects on the transport phenomena of electroosmotic flow.

2. Problem formulation

Steady, laminar, hydrodynamically and thermally fully developed electroosmotic flow inside a circular microtube with a rough surface is considered. The geometry of the problem is shown schematically in Fig. 1. The surface roughness is assumed to be approximately modeled by considering a corrugated wall of the form $r_w = R + R\varepsilon\sin(M\theta)$, where R is the mean radius of the microtube and M stands for the number of corrugations. Moreover, ε denotes the relative roughness. Based on the experimental data for the roughness amplitude of glass microchannels [37], this may take values of the order 0.01 assuming a channel of radius 10 μm. The electroosmotic-based microchannel heat sinks usually are made from silicon which may result in significantly rough surfaces; we here assume the values of up to 0.06 for ε based on the experimental data of Qu et al. [38]. Following the findings of Sadeghi et al. [39], the thermophysical properties are assumed to be computed based on the bulk mean temperature for the temperature-dependent influences to remain negligible. The zeta potential of the channel is not only constant and uniform but also is low enough to permit the usage of the Debye-Hückel linearization. In addition, it is assumed that the effect of temperature variations on the potential distribution within the EDL may be neglected [40]. The thermal features are analyzed considering the Joule heating effects and assuming the H1 thermal boundary conditions.

2.1. Electrical potential distribution

The combination of externally applied potential Φ and EDL potential ψ will constitute the electrostatic potential field φ in the channel. The former is only dependent upon the axial direction so that

$$\varphi(r,\theta,z) = \Phi(z) + \psi(r,\theta) \tag{1}$$

The relationship between the electrostatic potential and the net electrical charge density ρ_e is given by the Poisson equation:

$$\nabla^2\varphi = -\frac{\rho_e}{\epsilon} \tag{2}$$

where ε is the permittivity constant of the solution. In general, the connection between the electrostatic potential and the electric charge density should be described by the Nernst-Planck equations. However, at the hydrodynamically developed conditions, the spatial distribution of the electric charge density is expressed by the Boltzmann equation, despite the fact that it assumes the thermodynamic equilibrium [41]. This may be attributed to the orthogonality of the velocity vector and the ion concentration gradient at the fully developed conditions. Utilizing the Boltzmann distribution, the electric charge density for a solution containing N ionic species becomes [42]:

$$\rho_e = e\sum_{i=1}^{N} z_i n_{i0} e^{\left(-\frac{e z_i \psi}{k_B T_{av}}\right)} \tag{3}$$

where e represents the proton charge, z_i and n_{i0} are the valence number and concentration of the ith species at neutral conditions, respectively, k_B is the Boltzmann constant, and T_{av} is the average absolute temperature over the channel cross section. Substituting the charge density expression into the Poisson equation, and considering a constant voltage gradient in the z-direction, Eq. (2) reduces to the Poisson–Boltzmann equation

$$\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} = -\frac{e}{\epsilon}\sum_{i=1}^{N} z_i n_{i0} e^{\left(-\frac{e z_i \psi}{k_B T_{av}}\right)} \tag{4}$$

Eq. (4) is nonlinear and cannot be solved analytically; nevertheless, for small potentials, it can be linearized by replacing the exponential terms with their Taylor series and discarding all the terms of the order ψ^2 and higher. This approximation, first introduced by Debye and Hückel [3], has been shown to be valid for zeta potentials of up to 50 mV [39]. Making use of the electroneutrality conditions in the bulk where $\psi = 0$, the linearized form of the Poisson–Boltzmann equation in dimensionless form becomes

$$\nabla^{*2}\psi^* = \frac{\partial^2\psi^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial\psi^*}{\partial r^*} + \frac{1}{r^{*2}}\frac{\partial^2\psi^*}{\partial\theta^2} = K^2\psi^* \tag{5}$$

where $\psi^* = \psi/\zeta$, $r^* = r/R$, and $K = R/\lambda_D$ is the dimensionless Debye-Hückel parameter with $\lambda_D = \left(\sum_{i=1}^{N} n_{i0}e^2 z_i^2/\epsilon k_B T_{av}\right)^{-1/2}$ as the Debye length, a measure of the extent of EDL. The boundary conditions for the dimensionless electrical potential equation are as follows

$$\psi^* = 1 \quad \text{at} \quad r^* = 1 + \varepsilon\sin(M\theta) \tag{6}$$

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