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A contribution to the modelling of heat conduction for cylindrical composite conductors with non-uniform distribution of constituents

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ABSTRACT

This contribution deals with a heat conduction in a certain composite conductor with a deterministic structure, which is not periodic but in small regions of a conductor can be approximately regarded as periodic. The aim of this contribution is to formulate and apply two mathematical models for the analysis of heat transfer in skeletal rigid conductors. The formulation of the macroscopic mathematical models for analysis of heat transfer in these conductors will be based on the tolerance averaging technique (cf. monographs edited by Woźniak et al. (2008, 2010)). The general results are applied to the analysis of some special problems. It will be carried out validation of the obtained mathematical model by comparison of results from obtained tolerance-asymptotic model equations with results from finite elements method (Abaqus program).

1. Introduction

The object of consideration is a rigid skeletal heat conductor made of two families of thin cylindrical and radial elastic walls of constant width with mid-surfaces intersecting under the right angle. The regions situated between the walls are occupied by a homogeneous elastic matrix material. It is assumed that the cross-sections of the conductor perpendicular to the walls mid-surfaces represent a certain plane hetero-periodic structure (Fig. 1). For the sake of simplicity, the analysis will be restricted to the plane problems. The exact description of the conductor under consideration leads to equations with highly oscillating and discontinuous coefficients. The basic modelling problem is how to describe inhomogeneous material structure of the conductor by partial differential equations with functional but smooth and slowly varying coefficients.

The aim of the contribution is to derive and apply macroscopic mathematical models describing the heat conduction in skeletal cylindrical conductor which is not periodic but in small regions of a conductor can be approximately regarded as periodic. The conductor under consideration has deterministic micro-structure which is periodic along the angular axis and has smooth and slow gradation of effective properties in the radial direction. Since effective properties of this conductor are graded in space then we deal here with a special case of functionally graded material.

Many monographs and papers have been devoted to the modelling of heat transfer problems in micro-heterogeneous conductors. The analysis of the heat transfer in a hollow cylinder made of functionally graded material can be found for example in Hosseini et al. [4]. The heat transfer in micro-heterogeneous conductors is described by differential equation with highly oscillating and discontinuous coefficients. The best known averaging approaches are those based on the asymptotic homogenization cf. Bensoussan et al. [2]. Homogenization can be also realized using a concept of the micro-local parameters, cf. Matysiak and Yevtushenko [12]. For instance, in the paper of Matysiak and Perkowski [11] stationary two-dimensional heat transfer problem in periodically layered laminae was considered and homogenized model with micro-local parameters was used. In the paper of Kulchytsky-Zhyhailo et al. [7] the same model with micro-local parameters was applied for contact problems in a moving infinitely long punch on the boundary plane of vertically laminated half-space.

The formulation of averaged mathematical model of the skeletal conductor will be based on the tolerance averaging approach. This modelling technique describes the effect of the micro-structure size on the overall response of a composite structure. The general modelling procedures of this technique are given by Woźniak et al. [26,28].

The application of this technique for modelling and analysis of periodic composites and structures were presented in series

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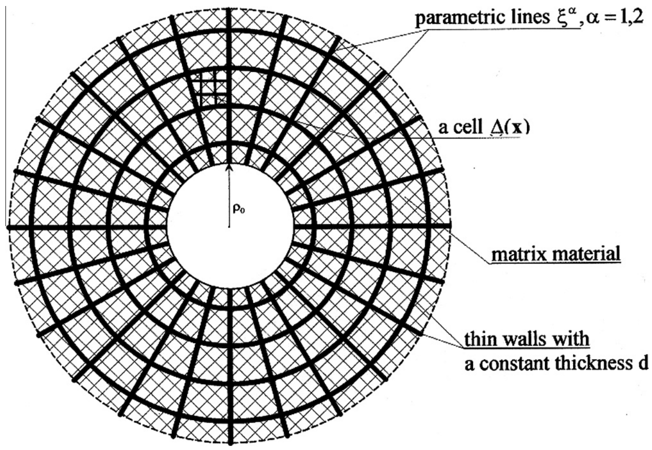

Fig. 1. A cross-section of analysed composite conductor.

papers, e.g. for laminates by Matysiak and Nagórko [10], for porous media by Dell'Isola et al. [3], for dynamic problems by: Baron [1], Jedrysiak and Woźniak [6], Michalak [13], Michalak et al. [17], Tomczyk [23], Woźniak and Wierzbicki [27]. We can mention here also a series of papers where problems of heat conduction were analysed, e.g. [9,8,19,21,24]. In the books edited by Woźniak et al. [26,28] the extended list of references on this subject can be found. The tolerance averaging technique was also adopted and applied to formulate mathematical models and to analyse various problems for functionally graded stratified solids, e.g. Jedrysiak and Radzikowska [5], Michalak [18], Michalak and Wirowski [16], Michalak [14,15], Ostrowski and Michalak [20], Rychlewska and Woźniak [22], Wirowski [25], Woźniak et al. [29].

In this paper governing averaged equations of the tolerance and asymptotic models for the heat conduction in skeletal conductor are derived. Application of these equations to special problems of heat conduction are also shown.

2. Preliminaries

By $\Omega \subset \mathbb{R}^3$ we shall denote a region in Euclidean space occupied by a conductor under consideration. We introduce a polar coordinate system $O\xi^1\xi^2\xi^3$ in $\Omega = \Pi \times I$, i.e.

$$\Omega = \{(\xi^1, \xi^2, \xi^3) \in \mathbb{R}^3 : (\xi^1, \xi^2) \in \Pi, \xi^3 \in I\}, \tag{1}$$

where $\Pi \subset \mathbb{R}^2$ is a plane and regular region. Throughout this paper indexes $\alpha, \beta, \ldots$ run over 1, 2, and a vertical line before the subscript α stands for the covariant derivative $\partial_\alpha \equiv \partial/\partial\xi^\alpha$ in the polar coordinate system. Summation convention holds for all aforementioned indexes. In the cross-section Π of the conductor perpendicular to the walls mid-surfaces (Fig. 2), parametric lines $\xi^\alpha, \alpha = 1, 2$, of a plane curvilinear coordinate system were assumed.

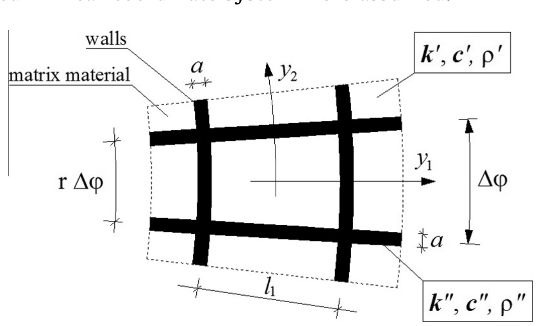

Fig. 2. A unit cell micro-structure.

The considerations are based on the well known Fourier theory of the heat conduction. The heat conduction properties are uniquely described by the second order heat conduction tensor $k^{\alpha\beta}(\cdot)$, specific heat scalar $c(\cdot)$ and density $\rho(\cdot)$. Let $\Theta(\cdot, t)$ be a temperature field in Π at time $t \in [t_0, t_1]$ and $f(\cdot, t)$ be the known intensity of heat sources which will be referred to as the thermal load. Under the aforementioned denotations the temperature field has to satisfy in Π the well known heat transfer equation

$$(k^{\alpha\beta}\Theta_{|\beta})_{|\alpha} - c\rho\dot{\Theta} = -f. \tag{2}$$

This direct description leads to the equation with highly oscillating coefficients, which is too complicated to be used in the engineering analysis of the heat conduction problems and numerical calculations.

3. Modelling concepts

Introduce the polar coordinate system $O\xi^1\xi^2$ so that the plane cross-section of the conductor $\Pi := \{(\xi^1, \xi^2) \in \mathbb{R}^2 : \xi^1 \in (R_1, R_2), \xi^2 \in (0, 2\pi)\}$. Assume in considered composite that the number of walls (inclusions) in radial and angular direction is n and m, respectively $(n^{-1}, m^{-1} \ll 1)$. Hence, $l_1 = (R_2 - R_1)/n$ and $\Delta\varphi = 2\pi/m$ are the cell dimensions according as coordinates ξ^1, ξ^2, respectively, and the unit cell

$$\Delta := \left(-\frac{l_1}{2}, \frac{l_1}{2}\right) \times \left(-\frac{\Delta\varphi}{2}, \frac{\Delta\varphi}{2}\right). \tag{3}$$

For any arbitrary cell $\Delta(\xi) := \xi + \Delta$ we introduce the orthogonal curvilinear coordinate system Oy_1y_2, which is local system with the origin at $\xi = (\xi^1, \xi^2) \in \overline{\Pi}_\Delta$, where

$$\Pi_\Delta := \left(R_1 + \frac{l_1}{2}, R_2 - \frac{l_1}{2}\right) \times (0, 2\pi) \subset \Pi \tag{4}$$

is so called *cell centres region*. Moreover for $\xi \in \overline{\Pi}$ we define

$$\Pi_\xi := \Pi \cap \bigcup_{\boldsymbol{\eta} \in \Delta(\xi)} \Delta(\boldsymbol{\eta}) \tag{5}$$

as a cluster of 4 cells having common sides. Denoting the diameter of cell $\Delta(\xi)$ by $l(\xi)$, we define a micro-structure parameter

$$l := \max\left\{\max_{\xi \in \overline{\Pi}_\Delta} l(\xi), \left(R_2 - \frac{l_1}{2}\right) \cdot \Delta\varphi\right\}, \tag{6}$$

and assume that l is sufficiently small compared to the smallest characteristic length dimension L_Π of Π $(l \ll L_\Pi)$.

In order to derive averaged model equations we applied tolerance averaging approach. This technique is based on the concept of tolerance and in-discernibility relations. The general modelling procedures of this technique and its basic concepts as a locally periodic function, an averaging operator, a tolerance parameter, a tolerance periodic function, a slowly varying function and a highly oscillating function are given in books [26,28].

We shall use in this contribution the concept of *locally periodic function* $f : \overline{\Pi} \to \mathbb{R}$ (cf. [2,28]) assumed in the form

$$\tilde{f}(\xi, \cdot) = f(\cdot)|_{\Delta(\xi)}, \quad \xi \in \overline{\Pi}, \tag{7}$$

where $\tilde{f} : \overline{\Pi} \times \Delta \to \mathbb{R}$ is periodic function with respect to coordinate $\xi \in \overline{\Pi}$.

Now, for the arbitrary locally periodic function $f(\cdot)$ the averaging operator over the cell $\Delta(\cdot)$ is defined by

$$\langle f\rangle(\xi) := \frac{1}{|\Delta|} \iint_{\Delta(\xi)} \tilde{f}(\xi, \mathbf{y})d\mathbf{y}, \tag{8}$$