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Marangoni abnormal convection heat transfer of power-law fluid driven by temperature gradient in porous medium with heat generation

Chengru Jiao [a], Liancun Zheng [a,*], Yanhai Lin [b], Lianxi Ma [c], Goong Chen [d,e]

[a] School of Mathematics and Physics, University of Science and Technology Beijing, Beijing 100083, China
[b] School of Mathematical Sciences, Huaqiao University, Quanzhou 362021, China
[c] Department of Physics, Blinn College, Bryan, TX 77805, USA
[d] Department of Mathematics and Institute for Quantum Science and Engineering, Texas A&M University, College Station, TX 77843, USA
[e] Science Program, Texas A&M University at Qatar, Education City, Doha, Qatar

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ABSTRACT

In this paper we investigate Marangoni convection heat transfer of power-law fluids in porous medium with heat generation. The convection is driven by a temperature gradient that the surface tension is a quadratic function of the temperature. A new heat transfer constitutive equation is proposed based on *N*-diffusion proposed by Philip and the abnormal convection–diffusion model proposed by Pascal in which we assume that the heat diffusion depends non-linearly on both the temperature and the temperature gradient with modified Fourier heat conduction for power-law fluid. The governing partial differential equations are reduced to ordinary differential equations by suitable similarity transformations. Approximate analytical solution is obtained using homotopy analytical method (HAM) which is compared with numerical ones for particular cases in good agreement. The transport characteristics of velocity and temperature fields are analyzed in detail.

1. Introduction

Marangoni convection, induced by surface tension gradient on the interface, is a very important physical phenomenon under microgravity conditions [1]. It has received much attention in recent years [2]. In the mid 1860s, Marangoni found the phenomenon that the natural convection dominated by liquid gravity gradually disappeared in a microgravity environment, whereas, at the interface of liquid, the surface tension plays a leading role and causes a surface tension gradient [3]. In 1978, Napolitano observed that there may be a dissipative layer in liquid–liquid or liquid–gas system, which is called Marangoni boundary layer [4]. According to the different origin, Marangoni effect is divided into the thermal effect of Marangoni (EMT) and the solute Marangoni effect (EMS) [5]. The EMT is mainly caused by the disequilibrium of the surface heat. The EMS is mainly caused by the imbalance of surface adsorption system.

Non-Newtonian fluids are found in many engineering applications where they exhibit some significantly different dynamic behavior from Newtonian fluids. For half a century, a considerable effort has been devoted to the studying of non-Newtonian fluids with the aim of predicting their complex flow, heat and mass transfer mechanisms. Various different constitutive equations were proposed, among which the power-law model is much attractive and has been widely used in many fields of application. Schowalter [6] and Acrivos et al. [7] firstly applied deduced the laminar boundary layer equations of power-law fluid flow over the semi-infinite flat plate. For an incompressible power-law fluid past a flat surface, its power-law shear rate-shear stress relation is expressed as $\tau = \mu \cdot \partial U/\partial Y|\partial U/\partial Y|^{n-1}$, where μ is the consistency and n is the power-law exponent of the fluid. The case $n = 1$ corresponds to a Newtonian fluid and the case $0 < n < 1$ is “power law” relation proposed as being descriptive of pseudo-plastic non-Newtonian fluids and $n > 1$ describes the dilatant fluid. Pop et al. [8,9] studied mixed convection heat transfer of power-law non-Newtonian fluids from a vertical surface, a modified Fourier heat conduction law was proposed, which take the effects of power-law viscosity on temperature fields into account. Zheng et al. [10,11] proposed a new heat transfer model by assuming that the temperature field is similar to the velocity field, the effects of power-law viscosity on heat conductivity are analyzed. In addition, Li et al. [12–14] numerically simulated the phenomenon of flow, heat transfer and diffusion of the power-law fluid in the circular tube. Lin et al. [15–18] studied the heat and mass transfer of steady laminar Marangoni convection driven by surface tension gradient using numerical method.

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[*] Corresponding author.
E-mail address: liancunzheng@ustb.edu.cn (L. Zheng).

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Nomenclature

A	positive constant, [–]
c	dimensional concentration, [–]
c_p	specific heat at constant pressure, [J kg^{-1} k^{-1}]
f	similar stream function, [–]
m	temperature power-law index, [–]
n	power-law index, [–]
P	dimensional porosity parameter, [–]
P_0	porosity parameter, [–]
Pr	Prandtl number, [–]
Q	dimensional heat generation parameter, [–]
Q_0	heat generation coefficient, [–]
T	temperature, [K]
T_∞	temperature of species far from the surface, [K]
t	dimensional temperature, [–]
U, V	velocity components along X and Y directions, respectively, [m s^{-1}]
u, v	dimensional velocity along x and y directions, respectively, [–]
X, Y	cartesian coordinates, [m]
x, y	dimensional cartesian coordinates, [–]

Greek symbols

δ_0	The minimum value of the surface tension, a positive constant, [N m^{-1}]
δ	Surface tension, [N m^{-1}]
η	Location similarity variable, [–]
θ	Temperature similarity variable, [–]
ρ	Density, [kg m^{-3}]
ψ	Stream function, [Ω^{-1} m^{-1}]

In 1961, Philip proposed a model for some special diffusion process as [19] $\partial C/\partial t = \nabla \cdot (AB)$, where C is the generalized concentration, t is time, A is a constant and B is a function of the concentration gradient ∇C. When $B = \nabla C$, we can get the classical diffusion equation (Fick's Law). Let $B = |\nabla C|^{N-1}\nabla C \ (N > 0)$, we obtain the so-called N-diffusion equation. Wu [20] and Wang [21] investigated a free boundary nonlinear problem for the N-diffusion equation, existence, uniqueness and analyticity results were established. Later, Pascal [22–24] presented a new convection–diffusion model by consider the nonlinear molecular diffusion for mass transfer in a two-phase system. In the model, the molecular diffusion is proposed to depending non-linearly on both the concentration and the concentration gradient as the form $B = C^m|\nabla C|^{N-1}\nabla C \ (N \geqslant 1)$..

This paper focused on Marangoni convection caused by the surface tension which is quadratic functions of the temperature. We proposed a new model for a constitutive relation of $B = (T - T_\infty)^m|\nabla T|^{n-1}\nabla T \quad (0 < n \leqslant 1)$. The effects of power-law fluid viscosity on temperature field is taken into account by assuming that the temperature field is similar to the velocity field [25–26], the governing partial differential equations are transformed into ordinary differential equations by using similarity transformations. Analytical solutions are obtained using Homotopy method (HAM) [27], the accuracy and effectiveness of analytical results are verified by numerical solutions. The effects of temperature power law index, the velocity power law index, the dimensional porosity parameter, the Marangoni Number and the dimensional heat generation parameter on the velocity and the temperature fields are graphically illustrated and analyzed.

2. Mathematical formulation

Consider two-dimensional, steady, laminar boundary layer flow of an incompressible power-law fluid in porous medium over a plate surface in the presence of surface tension due to temperature gradient (see Fig. 1 schematic of the physical system). It is assumed that the interface is not deformed with heat generation effect. Marangoni effect acts as a boundary condition on the governing equations for the flow. Taking the above assumptions into consideration, the governing equations are written as [5,28]

Continuity equation

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{1}$$

Momentum equation

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = \frac{\partial}{\partial Y}\left(v\left|\frac{\partial U}{\partial Y}\right|^{n-1}\frac{\partial U}{\partial Y}\right) - P_0(X)U, \tag{2}$$

Energy equation

$$U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \frac{\partial}{\partial Y}\left(\omega(T - T_\infty)^m\left|\frac{\partial T}{\partial Y}\right|^{n-1}\frac{\partial T}{\partial Y}\right) + Q_0(X)(T - T_\infty), \tag{3}$$

$$P_0(X) = \frac{v\phi}{k}X^a, \quad a = \frac{-m(3n+1) - (n-1)^2}{3mn + (n-1)(n-2)} \tag{4}$$

$$Q_0(X) = \frac{Q_0}{\rho c_p}X^b, \quad b = \frac{-m(6n+1) - (n-1)(2n-3)}{3mn + (n-1)(n-2)}. \tag{5}$$

where U and V are the velocity components in the X and Y directions, respectively. T is the temperature. $v\left|\frac{\partial U}{\partial Y}\right|^{n-1}$ is the kinematic viscosity, $\alpha = \omega(T - T_\infty)^m\left|\frac{\partial T}{\partial Y}\right|^{n-1}$ is the thermal diffusivity, ϕ is the porosity, k is the permeability of the porous medium, Q_0 is the heat generation coefficient, ρ is the density, c_p is the specific heat at constant pressure. T_∞ is the constant, denote the temperature of species far from the surface. Here we only consider the Pseudo-plastic fluid and Newtonian fluid. The dependence of surface tension on temperature can be expressed as

$$\delta = \delta_0 + \frac{\gamma}{2}(T - T_\infty)^2, \tag{6}$$

and

$$\gamma = \left.\frac{\partial^2\delta}{\partial T^2}\right|_{T=T_\infty}, \tag{7}$$

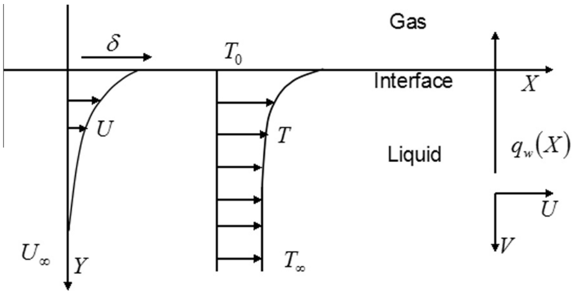

Fig. 1. Schematic of the physical system.