International Journal of Heat and Mass Transfer 92 (2016) 792–806

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

A new formulation of variable turbulent Prandtl number for heat transfer to supercritical fluids

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ARTICLE INFO

Article history:
Received 6 February 2015
Received in revised form 15 September 2015
Accepted 15 September 2015
Available online 30 September 2015

Keywords:
Turbulent Prandtl number
Reynolds analogy
Mixed convection
Supercritical pressure
Strong property variation

ABSTRACT

When a fluid at supercritical pressure approaches the pseudo-critical temperature it experiences a strong variation in physical properties putting applicability of various turbulent flow modelings in question. Earlier numerical calculations showed, without exception, unrealistic over-predictions, as soon as the fluid temperature approached the pseudo-critical temperature. The over-predictions might have been resulted either from an inapplicability of widely used turbulence models or from an unrealistic treatment of the turbulent Prandtl number (Pr_t) as a constant. Recent research, both numerical and experimental, indicates that Pr_t is very likely a function of fluid–thermal variables as well as physical properties, when the gradients of physical properties of a fluid are significant. This paper describes the procedure for a new formulation of Pr_t which varies with physical properties and fluid–thermal variables. The application of the variable Pr_t was surprisingly successful in reproducing the fluid temperature in supercritical fluids flowing in small-diameter vertical tubes ranging from 4.57 to 20 mm.

1. Introduction

An accurate estimation of the heat transfer rate or temperature of the coolant channel is essential for the development of a supercritical pressure water cooled reactor (SCWR) [1]. Methods for predicting the heat transfer rate to or from supercritical fluids flowing in a very narrow passage are not satisfactory and have yet to be established. The two kinds of fluid, water and carbon dioxide (CO_2), are mediums of interest and lot of works for the investigation are being conducted for applications in areas such as SCWR, Brayton cycle and compact printed circuit heat exchangers. A number of correlations for the prediction of the heat transfer rate in fluids at supercritical pressures have been proposed by various researchers, but most of them are applicable fluids in a forced convection regime, as shown in the review papers by Cheng and Schulenberg [2] and Pioro and Duffey [3]. The correlations available in literature predict the heat transfer rate with a reasonable accuracy in a forced convection regime; however, in a mixed convection regime, all of those correlations fail or partially succeed to produce accurate predictions, and the variation is so large that their application to the design needs to be very cautious.

Since most of the earlier works have been summarized by Pioro and Duffey [3], several selected recent works are introduced here. Efforts, both experimental and analytical, have been made to formulate a reliable correlation for a mixed convection heat transfer by researchers such as Watts and Chou [4], Jackson and Hall [5], Jackson et al. [6], Bae and Kim [7], Bae et al. [8], Bae [9] and Jackson [10]. Zhu et al. [11] investigated the heat transfer characteristics of steam–water flowing upward in tubes at sub- and super-critical pressures in the range of 13–30 MPa. Yang et al. [12] performed an experiment on heat transfer to supercritical water flowing in vertical annular channels, and evaluated four correlations against the data. Li et al. [13] reported recent experimental results from the supercritical water heat transfer test facility SWAMUP at Shanghai Jiatong University. Zhao et al. [14] reported experimental results from the same research group with different conditions only to reveal that the existing heat transfer correlations did not correctly reproduce the heat transfer rate.

In addition to the experimental efforts, a large number of numerical works have been performed to simulate the flow and thermal field in a fluid at supercritical pressures, and in doing so, the applicability of various turbulence models was examined. For both forced and mixed convection regimes, experimental and numerical investigations of the thermal and flow field at supercritical pressure was performed by Licht et al. [15]. They confirmed that for the simple case of deterioration, numerical simulations using the commercial CFD code Fluent offered a qualitative insight into changes in fluid temperature and turbulent velocities responsible for the axial evolution of the wall temperature. Cho et al. [16] examined three turbulence models, RNG k–ε, SST k–ω and one type

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2015.09.039
0017-9310/

Nomenclature

A^+	effective viscous sublayer thickness
C_μ	constant in the turbulent viscosity
$C_{\varepsilon 1}$, $C_{\varepsilon 2}$	constants in transport equation for e
c_p	specific heat
D	tube diameter
G	mass flux
G_k	production of turbulence due to buoyancy
h	enthalpy
k	turbulent kinetic energy
p	pressure
P_k	production of turbulence due to shear
Pr	Prandtl number
Pr_t	turbulent Prandtl number (variable)
$\mathrm{Pr}_{t,o}$	Pr_t before adjustment with additional functions
q	heat flux
r	radial coordinate
R	tube radius
Re	Reynolds number
T	temperature
u, v	velocity in x and r direction
u^+	non-dimensional u, u/u_τ
x	axial coordinate
y	distance from the wall
y^+	non-dimensional distance from wall, yu_τ/v
y^+_{TBL}	y^+ at the turbulent boundary layer edge (300)
$y^+_{r=0.8R}$	y^+ at $r = 0.8R$
$y^+_{u_{peak}}$	y^+ at $\partial u/\partial y = 0$
$\bar{a}$	Reynolds average quantity (a: dummy)
$\tilde{a}$	Favre average quantity (a: dummy)

Greek symbols

α_t	turbulent thermal diffusivity
β	volumetric expansion coefficient
ε	dissipation rate of turbulent kinetic energy
κ	von Karman constant
μ, μ_t	molecular and turbulent viscosity
v, v_t	molecular and turbulent kinematic viscosity
ρ	density
σ_k, σ_ε	model constants for turbulent diffusion of k, ε
σ_t	standard turbulent Prandtl number (=0.9)
τ_t	shear stress

Subscripts

e	effective (molecular + turbulent)
o	inlet
w	wall

of low-Reynolds number model, against the experimental data obtained for a tube and annulus with an equivalent hydraulic diameter of 4.4 mm, and reported that the performance of the three models was partially successful. He et al. [17] thoroughly investigated low-Reynolds number turbulence models and concluded that both the low Reynolds number k–ε models and the V2F models were able to capture the general trends of the interesting wall temperature behavior observed with an upward flow in some experiments with a fluid at a pressure just above the critical value, while the detailed variation of the wall temperature predicted using each model was rather different from that in the experiments. They also found that the effect on the heat transfer was almost entirely due to the shear production effect caused by the distortion of the mean flow as a result of the strong influence of the buoyancy. Zhang et al. [18] successfully reproduced using a modified version of a low-Reynolds turbulence model the data from a DNS calculation and an experiment by employing an algebraic flux model in calculating the turbulence production based on the buoyancy. However, its application to other conditions is still to be proven, and the calculation domain was too small to generalize the results. Zhang et al. [19] compared the experimental heat transfer data in supercritical fluids in a circular tube with the calculation results obtained by employing six different turbulence models and found that the Reynolds stress model (RSM) gave the best agreement with the experimental data, especially with the deteriorated ones. The result of RSM was not much different from that of RNG k–ε, and its applicability should be considered in parallel with the fact that it requires solving additional equations. Jaromin and Anglart [20] numerically performed a sensitivity analysis of the heated wall temperature and velocity distribution in the CFD simulation of the upward flow of supercritical water. They claimed that k–ω turbulence model successfully simulated the initial temperature peak near the inlet and onset of deterioration, but without the recovery of heat transfer from deterioration at the pipe exit. In summary, the numerical works performed thus far only to prove that all current turbulence models are applicable to the cases with limiting conditions. In this context, a break-through is needed to simulate the fluid thermal behavior in a fluid with severe property variation, and focus has been given to Pr_t rather than trying to improve the turbulence modeling.

Without exceptions all numerical works performed thus far struggled in simulating flow and thermal fields with strong property variation, that is, strong buoyancy or acceleration. Most of the turbulence modelings were developed based on an incompressible and constant-property flow. Afterwards, many efforts have been made to extend the models developed for constant property flows to variable property flows, however, they focused on high speed flow or compressible flows. Since the properties of fluids at supercritical pressure change with temperature than with pressure, a direct application of the theory developed for high speed or compressible flow to the cases of supercritical fluids should be very cautious. As expected, the application of the variable property (mainly density) version of turbulence modeling was not used in simulating a highly buoyant flow of fluids at supercritical flow, especially in reproducing fluid temperatures. Pr_t is a product of a pure perspective of similarity in appearance between the momentum and energy equations, and an assumption that their behavior would be the same or, at least, very similar to each other. Accordingly, it was treated as unity or an experimentally-obtained value slightly smaller than unity of 0.8–0.9. For most of fluid flows with barely varying fluid property, it has successfully worked and produced reasonable results, but never in the cases of strong property variation. In this regard, the author decided to revisit the Reynolds analogy, which connects the momentum and energy equations in the name of the Pr_t, and tried to find any possibility of extending it to be applied to the case of strong property variation.

In all numerical works introduced above, including earlier works not mentioned here, Pr_t was treated as a constant or function of Prandtl number. A constant Pr_t, which was not successful in predicting the heat transfer in a deterioration regime, does not seem to properly represent the physics in the supercritical fluid in a vertical tube under a heat flux high enough to cause deterioration. The Pr_t is highly unlikely to be a constant when the fluid properties experience substantial variations. Quarmby and Quirk [21] measured the eddy diffusivity in air flowing through a plain tube